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Robust filtering of weak signals from real phenomena

Filtraje robusto de señales débiles de fenómenos reales

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Abstract

In a large number of real-life scenarios it is required to process desired signals that are significantly immersed into background noise: tectonic signals from the entrails of the earth, signals coming from the far away cosmos, biometric telemetry signals, distant acoustic signals, noninvasive neural interfaces and so on. The purpose of this paper is to present the description of a robust and efficient platform for the real time filtering of signals deeply immersed in noise (rather weak signals) with rather different nature. The proposed strategy is based on two principles: the chaotic modelling of the signals describing the physical phenomena and the application of filtering strategies based on the theory of non-linear dynamical systems. Considering as a study case seismic signals, fetal electrocardiogram signals, voice-like signals and radio frequency interference signals, this experimental work shows that the proposed methodology is efficient (with mean squared error values less than 1%) and robust (the filtering structure remains the same although the phenomenological signals are drastically different). It turns out that the presented methodology is very attractive for the real time detection of weak signals in practical applications because it offers a high filtering precision with a minimum computational complexity and short processing times.

Keywords: Chaos, Nonlinear Filtering, Dynamic Systems, Kalman Filter, Weak signals, Real Signals.

Resumen

En un gran número de escenarios de la vida real se requiere procesar señales de interés que se encuentran muy inmersas en medio de ruido de fondo: señales tectónicas de las entrañas de la Tierra, otras provenientes del lejano cosmos, de telemetría biomédica, acústicas lejanas, interfaces neuronales no invasivas, etc. El propósito de este trabajo es presentar la descripción de una plataforma robusta y eficiente para hacer filtraje en tiempo real de señales muy inmersas en ruido (bastante débiles) y de naturaleza muy diferente. La estrategia propuesta se basa en dos principios: el modelado de las señales de los fenómenos físicos mediante procesos caóticos y la aplicación de estrategias de filtraje basadas en la teoría de sistemas dinámicos no lineales. Tomando como caso de estudio señales sísmicas, señales de electrocardiogramas fetales, señales de tipo voz y señales de interferencias de radiofrecuencia, este trabajo experimental muestra que la metodología es eficiente (error cuadrático medio menor al 1 %) y robusta (la estructura de filtraje, basada en filtro de Kalman, es invariante ante diferentes señales fenomenológicas). La metodología presentada resulta ser muy atractiva para aplicaciones prácticas para la detección de señales débiles en tiempo real por su alta precisión de filtraje con una mínima complejidad computacional y tiempos de procesamiento muy cortos.

Palabras clave: caos, filtraje no lineal, sistemas dinámicos, filtro de Kalman, señales débiles, señales reales.

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1. Introduction

Signal processing is basic for many areas of science and engineering. One of the fundamental stages for any signal processing algorithm is filtering, in other words, eliminating (filtering) background noise that accompanies the signal under study, prior to the corresponding signal processing.

There are various strategies for signal filtering, to counteract the effects of noise. The conventional filtering methodologies (based on statistical processing) have proven to be very rather effective when the signal-to-noise ratio (SNR) is greater than or equal to 1, i.e. SNR ≥ 1, or equivalently SNR ≥ 0 dB if it is expressed in decibels (namely, the magnitude of the noise less than or equal to the magnitude of the signal of interest).

In many practical applications, signals with noise are processed when the magnitude of the latter is greater than the magnitude of the signal of interest, i.e., SNR < 1. In order to filter signals in these situations, either conventional strategies have been adapted or novel methodologies have been created (based on iterative procedures, wavelets, etc.). The cost of successfully filtering signals under these conditions is that real time processing is slightly affected.

The filtering process is a great challenge when it is required to detect very weak signals (magnitude of the noise much greater than the magnitude of the desired signal, in other words, SNR « 1 or equivalently SNR « 0 dB), such as signals from distant solar systems, fetal cardiac activity, small seisms precursors of earthquakes, voice signals immerse in background noise, information signals of radiofrequency, noninvasive neural interfaces, among others. The issue of detecting weak signals is not really new, and it is possible to find a great number of publications (see [1–6] for citing only some references) that address this issue using various methodologies and for different phenomena. Techniques ranging from different filtering schemes (adaptive, time-frequency, FIR, IIR, among others), fuzzy logic, chaotic systems, stochastic resonance, to different decomposition strategies (empirical mode, wavelets, orthogonal signals, among others), are employed. The novelty for the case of the present work is the application of chaotic signals as models of real phenomena, based on the theory of deterministic nonlinear dynamic systems.

With a history of more than 50 years, the theory of dynamic systems [7–9] is one of the pillars of many scientific areas such as physics, automatic control, communications, etcetera; in particular, it is very relevant for filtering the famous Kalman filter (proposed by Rudolph E. Kalman in 1960), which enables a very precise filtering considering that the desired signal is a linear dynamic system under the influence of additive white Gaussian noise (AGWN), and from its invention until today there is a multitude of practical applications, both recent and old (for instance, [10, 11]).

How to achieve, based on the theory of nonlinear dynamic systems, an effective filtering strategy for weak signals from entirely different physical phenomena? One of the options that is explored in this work is modeling the phenomenological signals as signals generated by chaotic attractors, i.e., by nonlinear but deterministic dynamic systems. In this case the application of the concept of dynamic systems enables two very useful things: modeling real signals as formally deterministic processes, and applying all results about filtering based on the theory of dynamic systems.

Using chaotic models for real signals results original and rather effective, as shown later, even though the modeling of real phenomena by means of chaotic signals have been used during more than 50 years in areas such as seismology [1, 2, 12], statistical communication theory [13, 14], biomedical telemetry [15, 16], processing of submarine signals [3], and also in many areas related to applied physics [17].

This work presents an effective filtering (whose theoretical aspects were developed in [18, 19]) in the sense of a high precision in terms of very small values of the normalized mean square error (NMSE < 1 %). The normalization of the mean square error (MSE) is considered related to the variance of the phenomenological signal. On the other hand, filtering is also robust in the sense that, for input signals coming from different physical phenomena, both the structure and the precision (values of the NMSE) of the filter are practically invariant.

Chaotic modeling is very useful because nearly all algorithms of quasi-optimal filtering are characterized by having a high precision (very small value of the NMSE), and a very low computational complexity. Theoretical details and demonstration of these properties were developed in [18, 19], where the interested reader may review them. This work applies and extends to the practical field the nonlinear filtering ideas (published in [19]), presenting only the experimental details of physical scenarios with different phenomenology, specifically seismic signals, cardiac signals (ECG), vocal tract signals and radiofrequency interfering signals.

2. Materials and methods

The development of this work is centered in two basic elements. On one side, previously developed methods that provide a rigorous theoretical foundation and, on the other, MatLab code programming to create a test bench. The MatLab code is developed from the filtering equations described later. In the theoretical aspect, chaotic signals are employed both for filtering nonlinear dynamic systems and for modeling phenomenological signals. Since the idea is to obtain algorithms that may be implemented on a computational platform, it is important to establish the complexity of the nonlinear dynamic filtering in terms of operations and computational calculations.

2.1. Chaotic filtering and modeling

A chaotic process is defined from a set of ordinary differential equations and their corresponding parameters, i.e., is a deterministic process [17]. In the phase space, a chaotic process shapes an orbital trajectory, with the peculiar characteristic that none of all possible trajectories passes twice for exactly the same place [17]. In addition, a chaotic process is sensitive to changes in the initial conditions, i.e., two realizations of the same chaotic process whose initial conditions differ in an arbitrarily small value, are completely uncorrelated in the medium and long term [17].

Although being deterministic, a chaotic process generates realizations of processes that are described as stochastic, and it is precisely this deterministic-stochastic nature what is exploited to generate filtering strategies (with deterministic equations) and model physical processes (with stochastic realizations).

A vector chaotic process x(t) may be generated by means of the following ordinary differential equation (ODE):

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, t) \tag{1}$$

with initial condition $\mathbf{x}(t0) = \mathbf{x}0; \mathbf{F}(\cdot)$ is a time-varying vector function (represents the chaotic equations). In this case, $\mathbf{F}(\mathbf{x}, t)$ is considered taking as example the chaotic attractors of Rossler, Lorenz and Chua:

Rossler

$$\begin{aligned} x_{k+1} &= x_k + T_s(-y_k - z_k) \\ y_{k+1} &= y_k + T_s(x_k - 0.2y_k) \\ z_{k+1} &= z_k + T_s(0.2 - z_k(5.7 - x_k)) \end{aligned} \tag{2}$$

Lorenz

$$\begin{aligned} x_{k+1} &= x_k + T_s(10(x_k - y_k)) \\ y_{k+1} &= y_k + T_s(28x_k - y_k + x_k \cdot z_k) \\ z_{k+1} &= z_k + T_s(-\frac{8}{3}z_k + x_k \cdot y_k) \end{aligned} \tag{3}$$

Chua

$$\begin{aligned} x_{k+1} &= x_{k+1} + T_s[9.205(y_k - U(x_k))] \\ y_{k+1} &= y_{k+1} + T_s[x_k - y_k + z_3] \\ z_{k+1} &= z_{k+1} + T_s[-14.3y_k] \end{aligned} \tag{4}$$

where $U(x_k) = m_1x_k + \frac{1}{2}(m_0 - m_1)[|x_k + 1| - |x_1 - 1|$, $m_0 = -\frac{1}{7}$, $m_1 = \frac{2}{7}$ and T_S is the sampling time. The discrete Kalman filter (described a little further) is the filtering algorithm used, for this reason, (2)-(4) are presented in discrete form.

The initial conditions of the signal to be processed are unknown in the filtering block; this produces uncertainty (divergence) effects that can be mitigated including an additive «process noise» in Equation (1). Thus, the ODE becomes a stochastic differential equation (SDE), which gives rise to an n-dimensional Markovian stochastic process:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t)) + \varepsilon\xi(t) \tag{5}$$

where $\mathbf{f}(\mathbf{x}(t))$ is analogous to $\mathbf{F}(\mathbf{x}, t)$ in (1). The influence of an external weak source of white noise is denoted as $\xi(t)$; the intensities of the noise are given in form of the matrix $\varepsilon = [\varepsilon_{ij}]^{nxn}$.

When using the SDE (5) as a model for the chaos, the first strategy that immediately comes to mind is the nonlinear filtering of chaotic signals developed rigorously in [18, 19], which is based on the Stratonovich-Kushner equations (SKE) [7, 8] that can be used to describe the dynamic equation of the a posteriori probability density function (PDF) of the chaos x(t). Note that the time evolution of the a posteriori PDF for $x(t)$ is completely characterized by the SKE; however it does not have an exact analytical solution. One of the few exceptions is using a linear SDE, in other words, the well-known Kalman filter algorithm. Precisely, this is the reason why the nonlinear filtering algorithms are usually simplified making them quasi-optimal, or even quasi-linear.

The following question may arise: what are the advantages offered by the chaotic modeling for filtering weak signals? It turns out that the solution of the SKE for the dynamic ODE of the chaos (1), exhibits singularity properties when the solution is practically tuned with the deterministic chaos in (1), independently of the value of the SNR [18].

An important set of quasi-linear filtering algorithms apply the local Gaussian approximation (LGA) for the a posteriori PDF [8, 19], which results suitable for real time applications. Some of these algorithms are:

- Extended Kalman filter (EKF)
- Unscented Kalman filter (UKF)
- Quadrature Kalman filter (QKF)
- Gauss-Hermite quadrature filter (GHF)
- Conditionally optimal filter among others

Note that the difference between the algorithms based on the LGA, depends only on how the location of the instantaneous estimate of $x(t)$ is chosen. For the case of a high filtering precision, all algorithms that apply LGA [18] may be successfully approximated by means of the EKF, because the correct value of the filtering process and the reference point for the application of the Gaussian approximation are obviously very close.

Given a certain SNR, all these filtering algorithms have different precision and a completely different computational complexity, for a pre-established filtering quality. When selecting any specific filtering algorithm for a particular scenario, it is necessary to consider the NMSE in conjunction with the computational complexity, as possible criteria.

2.2. Computational complexity

In real world applications, the computational complexity of the quasi-linear algorithms is essential. For the particular case of the EKF, UKF, QKF and GHF algorithms, the computational complexity may be analyzed in terms of additions and multiplications, Cholesky decomposition, nonlinear propagation and Jacobian calculation. The evaluation with respect to these terms is shown in Table 1.

Table 1. Computational complexity

	EKF	**UKF**	**GHF**	**QKF**
Additions	8	50	25	25
Multiplications	15	77	33	40
Cholesky decomposition	1	2	2	2
Nonlinear Propagation	0	15	21	6
Jacobian calculation	1	0	0	0

It can be noted that the UKF exhibits a greater complexity, while the EKF is the less complex. The EKF might degrade due to the Jacobian calculation (evaluation of the partial derivatives), if the equations of the attractor are sophisticated. However, for the models of formulas (2)-(4), the filtering structure based on the EKF is the best choice. As will be shown later, when detecting real weak signals, a rather acceptable fidelity may be achieved in all practical cases using a filtering structure based on the EKF, which internally uses chaotic models of the type given by (2)-(4).

As an alternative to the quasi-linear algorithms of the EKF, where the linearization is updated instantaneously, a robust and low computational complexity solution may be searched for using a «fixed linearization» (with a linearization matrix predefined according to the specific problem under study) instead of instantaneous linearization. In fact, this means that the standard Kalman filter (SKF) [7–11] would be used instead of the EKF, and therefore, even though there would be a lower complexity, there would also be losses in the filtering precision. Nevertheless, it should be taken into account that the LGA of the a posteriori PDF assumes that all the components of the model are almost linear, and therefore the losses in precision may be reasonable.

Note that when the input data are variant, it is very common that the quasi-optimal filtering algorithms apply linearization strategies.

2.3. Multi-moment processing

To improve the filtering fidelity, it is required to take advantage of all the available information in the signal being processed. For this purpose, it makes sense to incorporate in the filtering methodology, applying the SKE equations, additional information (in different sequential time instants) of the received composite signal; in other words, it should be considered information in the form of blocks (at different time instants, i.e., multi-moment processing). The multi-moment algorithms are implemented through the generalization of the SKE using multi-moment data.

The multi-moment filtering algorithms are slightly practical for real time implementations, since the delay due to the processing of samples from different time instants is significant. To reach a compromise between complexity and increase of filtering precision, it is reasonable to consider processing only two adjacent samples. This processing is known as two-moments regime (2MM), which is a special case of the multi-moment filtering and may be reviewed in detail in [18, 19].

In the 2MM regime, two samples coming from two time instants (not instantaneous processing) are processed during every filtering cycle, and consequently the correlation coefficient between the two adjacent samples (denoted as ρ) is a design parameter. The advantage of considering the 2MM regime, is that the benefits of the multi-moment processing can be obtained, practically without significant delays.

In the one-moment (1MM) regime, in which one sample from a single time instant is processed during each cycle (instantaneous processing), there is no increase in the filtering precision, and this is precisely the type of processing that characterizes the EKF and its variants previously listed.

2.4. Simulation model

The practical implementation of the proposed design methodology, was carried out developing a simulation test bench based on MatLab.

The methodology is constituted by two elements:

1) A nonlinear dynamic filtering structure based on the Kalman filter (EKF or SKF according to the case).

2) Appropriate modeling of the real weak signal, congruent with the filtering structure to be used (EKF or SKF).

A discrete version of the Kalman filter is used for simulation by means of MatLab. Such discrete version is now described. The state dynamics in a discrete system is given by:

$$\begin{aligned} x_{k+1} &= f(x_k) + \varepsilon_k \\ y_k &= s(x_k) + n_{0ks} \end{aligned} \tag{6}$$

where x_k represents the state of the system, and y_k is a measurement of the state of the system, $\{n_{0k}\}$ and $\{\varepsilon_k\}$ are independent Gaussian white noise processes with zero mean and covariance matrices

$$E[n_{0k}n_{0k}^T] = N_{0k} \;\; and \;\; E[\varepsilon_k \varepsilon_k^T] = Q_k$$

respectively, Q_k denotes process noise and measuring noise. The prediction and correction cycles of the Kalman filter are given by:

$$\begin{aligned} &Prediction \begin{cases} \hat{x}_{k+1}^- = f(x_k^+) \\ P_k^- = A_k P_k^+ A_k^T + Q_k \end{cases} \\ &Correction \begin{cases} G_k = P_k^- H_k^T [H_k P_k^- H_k^T + N_{0k}]^{-1} \\ \hat{x}_k^+ = \hat{x}_k^- + Gk[y_k - s(x_k^-)] \\ P_k^+ = P_k^- + G_k H_k P_k^- \end{cases} \end{aligned} \tag{7}$$

where G_k is the Kalman gain, $\hat{x}_k^-$ is the a priori estimate of the state at the k-th update cycle, $\hat{x}_k^+$ is the a posteriori estimate of the state at the k-th update cycle, P_k^-, P_k^+ are, respectively, the a priori and the a posteriori estimations of the covariance matrices at the k-th stage, A_k is the linearization matrix (or state-transition matrix) and H_k is the matrix that indicates the relationship between the measurement and the state vector at the k-th cycle, assuming absence of noise. For the case of the SKF, Ak is a fixed matrix in every cycle, while for the EKF the matrix is updated in each cycle by means of the Jacobian calculation:

$$A_k = \frac{\partial f(x_k)}{\partial x_k} \tag{8}$$

When using the EKF, the structure of the filter is given by (7); (2), (3) or (4) are used as the function $f(x_k)$, and the linearization is given by (8). Here, the real weak signal is modeled using any of the chaotic processes (2)-(4), i.e., it is analyzed which of the components (x, y or z) of (2), (3) or (4) is the most appropriate to be used as model. For this purpose, the sampling time (T_s) of the discrete chaotic equations is first modified, until reaching a coincidence between the time variations of the component of the selected chaotic attractor and the real signal (make the time scales as close as possible). Second, the desired signal is normalized with respect to the mean and the variance of the component of the attractor. It has been demonstrated in [18, 19], that the x component of the three chaotic attractors (2)-(4) is the more appropriate to model experimental signals. After carrying out a similar modeling analysis, it was found that the same criterion is applicable for the phenomenological signals of this work.

When using the SKF, the matrix A_k is fixed and the modeling of the signal should be reflected precisely in A_k. With this purpose, the MatLab System Identification Toolbox (SIT) [20] is used. Such tool is based on the theory of systems identification [21]. To identify the real signal (seismic, ECG, voice-type, RFI), the MatLab SIT analyzes its spectral properties, and gives a constant matrix as the model.

The experimental part of the next section shows that it is possible to use such matrix precisely as a fixed linearization matrix in the filtering structure given by (7), which really is only an approximation of the instantaneous linearization procedure required for the quasi-optimal filtering when the LGA is used.

To make a fair comparison with the EKF of dimension 3 according to formulas (2)-(4), a tridimensional SKF is designed. The «ident» command is used to obtain matrix Ak by means of the MatLab SIT. The identification (of the signal without noise) is made selecting the option «state space models» [7, 22] for the three-dimensional case. The program provides three estimation options (subspace method, regularized reduction and minimization of the prediction error), and at the end indicates the confidence percentage of each option. It was found experimentally that the option of minimization of the prediction error offers the best confidence percentage, for the estimation of the matrix.

In the methodology proposed here, the systems identification is precisely a modeling of the real signal. The identification is made for a vector (the largest possible) of the real signal (without noise). Assuming that the signals under study are stationary, the system identification matrix may be considered as the fixed linearization matrix A_k and, therefore, be a signal model for any other vector of the same phenomenological signal. When using the identification matrix in the structure of the SKF, a processing with a priori information and experimental data is achieved.

The 2MM regime is employed to achieve a greater filtering fidelity. As it is commented in [19], the quasi-optimal solutions (in this case for multi-moment algorithms) are based in some heuristic that may incorporate previous knowledge and/or structures. If this is the case, the 2MM regime utilized in this work has the form:

$$
\begin{array}{rl}
\hat{x}_{k+1}^{-} = f(\hat{x}_k^{+}) & {}_2\hat{x}_{k+1}^{-} = f({}_2\hat{x}_k^{+}) \\
P_k^{-} = A_k P_k^{+} A_k^{T} + Q_k & {}_2P_k^{-} = {}_2 A_k P_k^{+} (1-\rho^2) {}_2A_k^{T} + Q_k \\
G_k = P_k^{-} H_k^{T} [H_k P_k^{-} H_k^{T} + N_{0k}]^{-1} & {}_2G_k = {}_2P_k^{-}\, {}_2H_k^{T} [{}_2H_k {}_2P_k^{-}\, {}_2H_k^{T} + N_{0k}]^{-1} \\
\hat{x}_k^{+} = \hat{x}_k^{-} + G_k[y_k - s(x_k^{-})] & {}_2\hat{x}_k^{+} = {}_2\hat{x}_k^{-} + {}_2G_k[y_k - s({}_2x_k^{-})] \\
P_k^{+} = P_k^{-} + G_k H_k P_k^{-} &
\end{array} \quad (9)
$$

where the subscript 2 of the left side of each variable denotes a 2MM variable. For this filtering algorithm, the only difference between the left and right columns of the structure given by (7), is that the last operation of the correction cycle does not exist; both columns operate in parallel. In (7), the output of the filter is $\hat{x}_k^{+}$, while in (9) is ${}_2\hat{x}_{k+1}^{-}$. In the 2MM regime, in the column of the right side in the last operation of the prediction cycle, the a priori estimate of the covariance matrix is calculated taking into account the correlation coefficient ρ of the two samples. Observe that the structure (9) may operate both for the EKF and the SKF, following the observations described in previous paragraphs.

Next section experimentally shows the efficiency of the filtering proposed here, considering signals of significantly different nature, such as, seismic signals, fetal electrocardiographic (FECG) signals, voice-type signals and radiofrequency interfering signals (RFI). Such signals may be considered chaotic [1, 2, 12, 15, 23, 24]

3. Results and discussion

The following results show the filtering by means of SKF and EKF, both for the 1MM and 2MM regimes. In the 2MM regime, the ρ parameter determines a different fidelity in the filtering. If $\rho = 0$, this corresponds to the 1MM regime (without increase in the fidelity). If $\rho = 1$, see formula (9), this corresponds to a singularity condition with a covariance matrix equal to zero (fidelity tends to $+\infty$)). It was chosen $\rho = 0.85$ for a homogeneous analysis of the results.

The figures show the overlapped curves of the original signal (without noise), and the signal after being filtered with the filtering scheme and the corresponding regime indicated in each figure. The figures only show a threshold case of weak signal when the $SNR =$ ˘$3dB$. Manipulating N0 to analyze different thresholds of weak signals ($SNR \leq 0dB$), Tables 2-5 show the performance of the SKF and EKF under the 1MM and 2MM regimes, in terms of the NMSE (described in the paragraphs of the introduction). The average times (in seconds) required to process 5000 samples for each of the phenomena under study and their corresponding filtering, are also shown.

For both the SKF and EKF, an imperfect modeling of the phenomenological signal is carried out (there is a certain degree of uncertainty in the initial conditions for the filtering); as a consequence, a noise value of weak process, see Q in (7) and (9), should be included in the filtering structure (value of Q indicated in tables 2-5).

No results of the SKF are presented for the seismic signals, since it was not possible to obtain the corresponding fixed linearization matrix (systems identification matrix) because such signals have a very limited duration for an adequate spectral analysis by means of the SIT.

3.1. Fetal electrocardiographic (FECG) signals

The experimental data were obtained from the PhysioNet [25] database. The signal for this experiment corresponds to the heart of a fetal product, at week 36 of pregnancy. For a $SNR =$ ˘$3dB$, Figure 1 shows the original signal and the signal filtered using the EKF 1MM with the x component of Rossler as model. Table 2 shows the complete results.

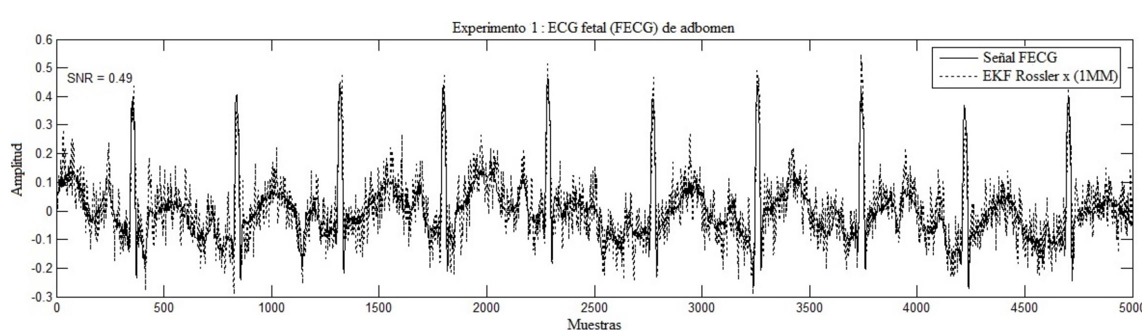

Figure 1. Signals in experiment 1

Table 2. Results of the NMSE for experiment 1

SNR	0 dB	–3 dB	–10 dB	Procesamiento time
SKF $Q = 0.04$ (with linearization matrix A_k)				
1MM	0.0025	0.0037	0.0078	0.43 s
2MM	0.0021	0.0032	0.0065	0.89 s
EKF Rossler x $Q = 0.21$				
1MM	0.0026	0.0040	0.0098	1.825 s
2MM	0.0023	0.0036	0.0079	3.503 s
EKF Lorenz x $Q = 0.42$				
1MM	0.0029	0.0042	0.010	1.782 s
2MM	0.0023	0.0034	0.0083	3.59 s
EKF Chua x $Q = 0.075$				
1MM	0.0034	0.0053	0.015	1.812 s
2MM	0.0026	0.0042	0.012	3.61 s

3.2. Experiment 2. Voice-type signals

Sustained sounds of vowels (vowel «o») were used for this experiment. This type of signals are utilized in voice synthesis procedures [23]. Figure 2 shows with solid line the sustained sound of the vowel «o» (recorded during 5 seconds at 22050 Hz), and with dotted line the signal filtered using the SKF 2MM with its matrix evaluated by means of SIT. The results (very similar to the previous experiment) are shown in Table 3. For this experiment none of the components of the Lorenz attractor resulted adequate for modeling voice-type signals.

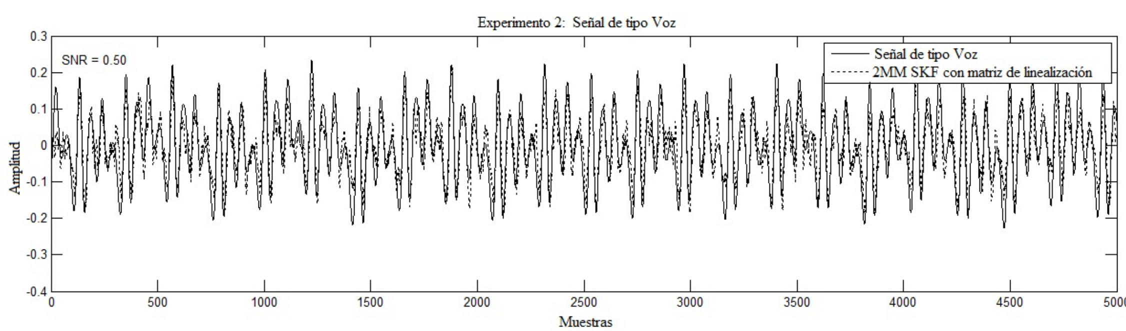

Figure 2. Signals in experiment 2

Table 3. Results of the NMSE for experiment 2

SNR	0 dB	–3 dB	–10 dB	Processing time
SKF Q = 0.0081 (with linearization matrix A_k)				
1MM	0.0025	0.0037	0.0079	0.47 s
2MM	0.0015	0.0024	0.0053	0.95 s
EKF Rossler x Q = 0.23				
1MM	0.0029	0.0044	0.0124	1.792 s
2MM	0.0027	0.0039	0.011	3.611 s
EKF Chua x Q = 0.76				
1MM	0.0031	0.0048	0.0137	1.81 s
2MM	0.0025	0.0043	0.0130	3.58 s

3.3. Experiment 3. Seismic signals

A MatLab simulator based on the seismic models reported in [26] was used for this experiment. For an $SNR = ˘3dB$, Figure 3 shows a seismic signal and its filtered version obtained using the EKF 2MM with the x component of the Rossler model. The complete results are shown in Table 4. For the seismic signal it was not possible to obtain the linearization matrix, and consequently the SKF is not reported for this case.

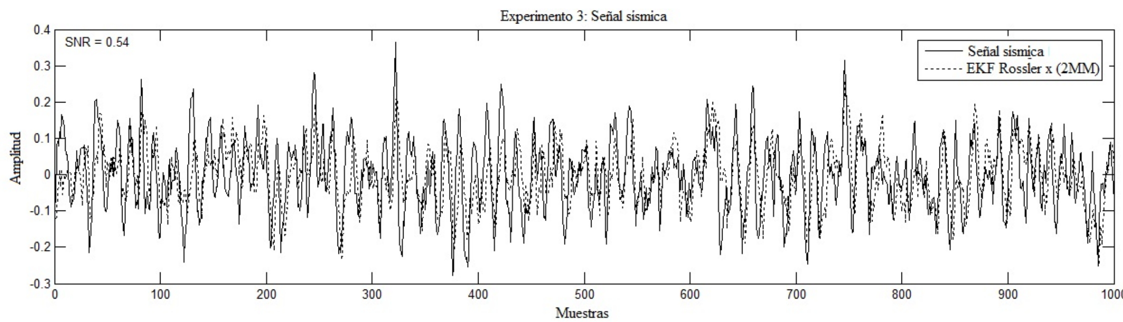

Figure 3. Signals in experiment 3

Table 4. Results of the NMSE for experiment 3

SNR	0 dB	-3 dB	-10 dB	Processing time
EKF Rossler x Q = 0.35				
1MM	0.0048	0.0074	0.0178	1.79 s
2MM	0.0047	0.0073	0.0135	3.53 s
EKF Lorenz x Q = 0.135				
1MM	0.0058	0.0093	0.0245	1.807 s
2MM	0.0054	0.0081	0.0187	3.62 s
EKF Chua x Q = 0.135				
1MM	0.0057	0.0095	0.029	1.816 s
2MM	0.0051	0.0084	0.023	3.65 s

3.4. Experiment 4: RFI signals

This experiment considers the RFI generated by computer equipment [24,27], which affects the transmission of desired information signals. For an $SNR = ˘3dB$, Figure 4 shows the RFI signal and its filtered version obtained using the SKF 1MM, with the linearization matrix evaluated by means of the SIT. Table 5 shows the complete results.

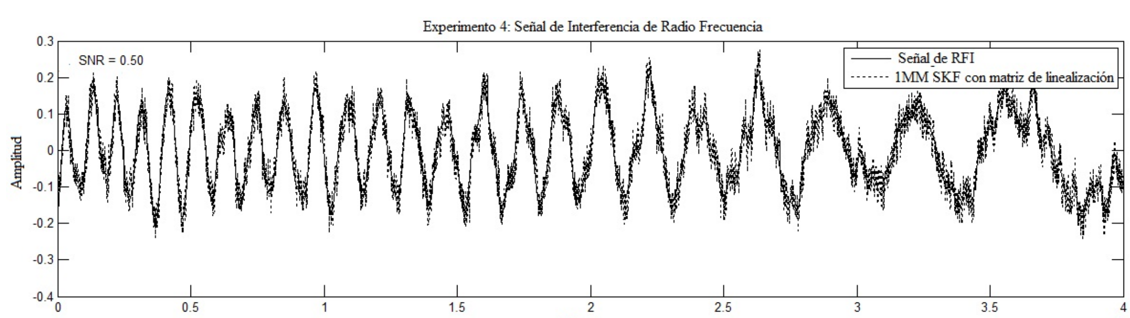

Figure 4. Signals in experiment 4.

Table 5. Results of the NMSE for experiment 4

SNR	0 dB	-3 dB	-10 dB	Processing time
SKF Q = 0.02 (with linearization matrix A_k)				
1MM	0.0018	0.003	0.0098	0.51 s
2MM	0.0015	0.0025	0.0085	0.92 s
EKF Rossler x Q = 0.2				
1MM	0.0026	0.005	0.019	1.872 s
2MM	0.0023	0.0036	0.011	3.9 s
EKF Lorenz x Q = 0.6				
1MM	0.0023	0.0032	0.04	1.76 s
2MM	0.0016	0.0027	0.0083	3.81 s
EKF Chua x Q = 0.4				
1MM	0.0034	0.0053	0.035	1.86 s
2MM	0.0026	0.0042	0.029	3.77 s

From the tables it is observed that the 2MM method shows a slightly better NMSE. All the filtering methodologies presented are rather effective, because they are characterized by a very low value of the NMSE. In the scenario of $SNR = \breve{}10dB$ (an extremely weak signal), it is virtually impossible to visually distinguish (it is not shown in figures due to space issues) the desired signals from the noise; nevertheless, the NMSE has values around 1 % for the filtering by means of SKF and EKF, both for the 1MM and 2MM methodologies.

It should be taken into account that the 2MM methodology consumes more time, compared to the 1MM methodology, although it is not more than twice the time. The filtering by means of SKF is (almost 3 times) faster, because there is no linearization process. The processing times together with the complexity and the filtering fidelity, should be the selection criteria to choose the adequate filtering algorithm for each particular implementation.

The SKF with fixed linearization matrix (modeling the signals of interest by means of a system identification matrix) shows the best results, which indicates that for processing with quasi-linear filtering algorithms, the influence of the spectral properties of the input data prevails over the influence of the non-Gaussian statistics. The values of NMSE obtained in the simulations are very similar for filtering the different signals, and consequently, it is not really determinant in practice which particular model of chaotic attractor or linearization matrix (obtained from the SIT) is applied.

Why does this occur? All the chaotic attractors that have been employed to model the real signals, generate chaos as a quasi-deterministic stochastic process. For this reason, all the aforementioned quasi-optimal filtering algorithms (including the EKF and its modifications) which apply the idea of chaotic modeling, operate in a regime very close to the singularity, i.e., the shape of the a posteriori PDF is concentrated around the a priori PDF of the desired signal, independently of the SNR value [18,19], and that precisely enables obtaining rather low values of the NMSE for very weak signals (SNR smaller than 0 dB and up to –10 dB). Therefore, for high fidelity filtering, the linear term of the Taylor series expansion for the quasi-linear algorithm [8,9,22] is significantly more determinant than the terms related to the nonlinearities (Jacobian matrix, etc.), i.e., the linear approximation is sufficient.

4. Conclusions

It has been proposed a simple and robust filtering structure based on the Kalman filter, for processing weak signals. This structure enables the incorporation of the 2MM regime, which improves the filtering precision.

For non-Gaussian phenomenological signals, depending on the specific scenario, requirements of computational complexity and acceptable error, it may be employed as model for the real signal either the EKF using chaotic signals, or the SKF with a fixed linearization matrix; in other words, the MatLab SIT may be used to evaluate the systems identification matrix, which is then used as model of the real signal. This enables a «significant degree of freedom» for the design of the filtering block.

The experimental results show great precision in filtering weak signals for all scenarios considered here, and given the rather diverse nature of such scenarios, most likely it may be applied in other scenarios (future work).

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