Scientific Paper / Artículo Científico

UNIVERSIDAD POLITÉCNICA SALESIANA ECUADOR
INGENIUS
Revista de Ciencia y Tecnología
https://doi.org/10.17163/ings.n29.2023.03
pISSN: 1390-650X / eISSN: 1390-860X

Calculation of abrasive wear speed in straight and helical gears with evolving profile, using a Matlab GUI

Cálculo de la velocidad de desgaste abrasivo en engranajes de dientes rectos y helicoidales con perfil evolvente, utilizando una GUI de Matlab

José Miguel Mena Chavarrea[1,*], José Antonio Granizo[2],
Eduardo Segundo Hernández Dávila[2], Mario Efraín Audelo Guevara[3]

Received: 11-02-2022, Received after review: 20-10-2022, Accepted: 09-11-2022, Published: 01-01-2023

Abstract

The present investigation originated under the objective of designing a software with Matlab that allows calculating the abrasive wear rate of spur and helical gears with involute profile. Where, the unstudied generalities about the design of gears have generated negative consequences in functionality and have generated catastrophic failures that have produced unexpected stoppages of mechanical processes, for which understanding their causes becomes essential. Considering, for the calculation of the abrasive wear of the tribological pairs in heavy machinery, it will depend on the degree of grinding or size of the abrasive particles, the hardness, the material of the base gear, the lubrication regime and the geometric conditions that determine the nature of tribological contact, as well as the time and speed of use of heavy machinery. Initially, the theoretical environment that makes up the study based on the main components such as: gears, lubrication, steel, heat treatments and finally Matlab is established.

Resumen

La presente investigación se originó con el objetivo de diseñar un *software* con Matlab que permita calcular la velocidad de desgaste abrasivo de engranajes rectos y helicoidales de perfil evolvente. Donde, las generalidades no estudiadas sobre el diseño de engranajes han generado consecuencias negativas en la funcionalidad y fallas catastróficas que han producido paras imprevistas de procesos mecánicos, por lo cual entender las causas de los mismos se vuelve fundamental. El cálculo del desgaste abrasivo de los pares tribológicos en maquinaria pesada dependerá del grado de trituración o tamaño de las partículas abrasivas, la dureza, el material del engrane base, el régimen de lubricación y las condiciones geométricas que determinen la naturaleza del contacto tribológico, así como el tiempo y velocidad de uso de la maquinaria pesada. Inicialmente, se establece el entorno teórico que compone el estudio basado en los principales componentes como engranajes, lubricación, el acero, los tratamientos térmicos y, al final, Matlab.

[1,*]Tecnología Superior en Electromecánica, Instituto Superior Tecnológico Cotopaxi, Ecuador.
Corresponding author ✉: jmmenac@istx.edu.ec
[2]Grupo de investigación Ciencia del Mantenimiento, Escuela Superior Politécnica de Chimborazo, Ecuador.
[3]Ingeniería Automotriz, Facultad de Mecánica, Escuela Superior Politécnica de Chimborazo, Ecuador.

Suggested citation: Mena Chavarrea, J. M.; Granizo, J. A.; Hernández Dávila, E. S. and Audelo Guevara, M.E. “Calculation of abrasive wear speed in straight and helical gears with evolving profile, using a Matlab GUI,” *Ingenius, Revista de Ciencia y Tecnología*, N.° 29, pp. 32-45, 2023, doi: https://doi.org/10.17163/ings.n29.2023.03.

Next, the mathematical models of Archard and Kraglesky were established, which were adapted to the circumstances of the problem and based on this information, a software capable of calculating the rate of abrasive wear was formulated using a graphical Matlab user tool; We proceed to execute the manual calculation with the determined variables taking experimental data obtained from previous investigations and that comply with the variables determined for the execution of the mathematical process, given that the results generated through the software can be tabulated, graphed in Excel and verified. with manual mathematical development. Looking for the comparison, differentiation and trend of the behavior of the wear rate, varying the parameters involved in the process using the two mathematical models proposed.

Keywords: Gears, Abrasive, Wear, GUI, Matlab, Wear rate

Seguidamente, se establecieron los modelos matemáticos de Archard y Kraglesky que fueron adaptadas a las circunstancias del problema y en función de esta información formular un *software* capaz de calcular la velocidad de desgaste abrasivo mediante una herramienta gráfica de usuario de Matlab. Se procede a ejecutar el cálculo manual con las variables determinadas, tomando datos experimentales obtenidos de investigaciones previas y que cumplan con las variables determinadas para la ejecución del proceso matemático, dado que los resultados generados a través del software puedan ser tabulados, graficados en Excel y comprobados con el desarrollo matemático manual. Buscando la comparación, diferenciación y tendencia del comportamiento de la velocidad de desgaste, variando los parámetros involucrados en el proceso utilizando los dos modelos matemáticos planteados.

Palabras clave: engranajes, desgaste, abrasión, GUI, Matlab, velocidad de desgaste

1. Introduction

At present, gears have a great importance as transmission elements in mechanical systems, since they are mechanical pieces found in any industry as main machine elements that generate force and movement. The degree of wear of the gears depends on the work carried out by the equipment, the grain size and composition of the abrasive, the working power, the design load, and the efficiency and reliability of the process. These are major factors that should be considered to determine the costs of executing corrective and preventive maintenance.

Detecting failures in a timely and efficient manner constitutes one of the most important challenges associated to predictive maintenance. Unexpected failures may affect the integrity and reliability of equipment through unscheduled stops, reduction of useful life, high costs of corrective maintenance and low quality of products [1].

Studying the wear process of gears is increasingly important, because they are the mechanical transmission elements mostly used in different fields and under different working conditions, from tiny gears used in watches or as part of any machine independent of its size.

In industry, wear is one of the most frequent problems that appear in systems that contain gears, regardless of the work they perform. The abrasives are elements that are found in the environment and are typical of heavy machinery elements, such as walking equipment, agricultural machinery, construction machinery, mining industry machinery, etc. The abrasive particles are a determining factor to significantly reduce the useful life of the gear.

The most important aspects to be considered in the design of the gear teeth profile, are related to the load capacity, transmission error, pressure angle, wear and failure analysis [2].

The lack of information and review of the analysis criteria for designers and for people responsible for machinery maintenance, whose main object of design are spur and helical gears with involute profile which are continuously subject to abrasive elements, has generated that mechanisms get damaged with the consequent unnecessary stops, and even the no implementation of laboratory research processes that are extremely delayed and costly. The abrasive found in the environment directly affects the performance of mechanical systems, since it generates catastrophic failures that produce losses to the industrial processes of companies. The parameters that have influence are grain size, abrasive hardness, quantity of abrasive and work carried out, which reduce the performance of industrial processes and the production.

Hence, it is necessary to study them to try to address the problems that may appear in gears. Thus, the wear, the abrasive and the failures are related aspects that will be the case study here, to determine the wear rate depending on the abrasive and on the gear material. The substitution of 90% of the gearwheels is due to an efficiency loss as a consequence of the wear of the teeth; in general, the following mechanisms are used to tackle this phenomenon:

- Optimization of geometrical parameters according to the design.
- Improvement of the quality of the teeth surface and of the assembly of the parts
- Appropriate selection of the material and of the parameters of the thermal treatment.
- Appropriate selection of the lubricant and optimization of the lubrication process [3].

Hence, it follows that there will be a greater chance that a hydrodynamic film is formed in the zone of the gear pole, and if lubricant is present in the tribological process, this zone will have less wear in the toothed transmissions.

The present work proposes the usage of a graphical user interface to carry out the theoretical calculation of the wear rate in gears with involute profiles, to provide the designer with tools that enable to predict the behavior of the tribological system, evaluate the validity of its design theories and generate timely maintenance plans.

Taking into account the approximations made, it is considered the process based on a scheme that uses vibrations to update a wear prediction model. First, it is developed a dynamic model of a system of spur gears to generate realistic vibrations, which enables a quantitative study of the effects of wear on the surface of the gear teeth. The sliding speed and the contact forces of the model are used in combination with the known Archard wear model, to calculate the wear depth at each contact point in the mesh. Since the wear coefficient in the model is not constant during the wear process (and in any case it is difficult to estimate initially), the vibrations measured are compared with the vibrations generated by the model, to update the coefficient when it is detected a deviation in the predictions [4].

1.1. Tribology

The term tribology comes from the Greek terms «tribos» and «logos», which mean friction and study, respectively. Hence, this term is used to designate the science that studies the surfaces with relative movements between each other.

For this reason, terminologies such as friction, wear of the different surfaces and the presence or absence of lubricant between the parts in contact, are essential to obtain machines and processes with less energy loss, avoid long stop times that limit the efficiency of the production activity, improve the life cycle of machines and, above all, have available a reliable tool to generate good repair and maintenance practices [5].

1.2. Mechanical contact

Figure 1 shows the apparent area corresponding to the entire surface of the parts in contact, and the real area, which considers that all surfaces have rough points that cause that the contact occurs only at the points of coincidence of the corresponding crests of each of the surfaces involved in the movement [6].

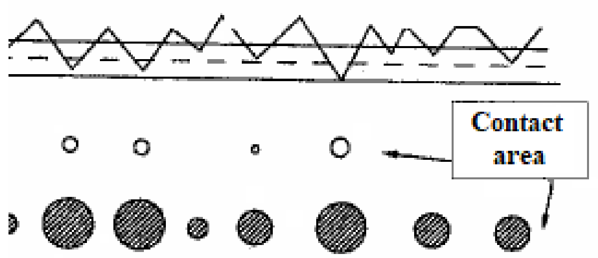

Figure 1. Apparent contact area [6]

1.3. Wear

Is the process of destruction and detachment of material that occurs between the surfaces of the bodies, revealed as accumulation of deformations and variation of the initial dimensions of the corresponding object [7].

1.3.1. Abrasive wear

This wear mechanism is characterized by the presence of hard particles that interact with surfaces that slip against each other. As it is determined in Figure 2, under this system it is important to characterize that this type of wear causes imperfections and microbreaks, due to the action of extremely hard and small particles, compared to the base surface [8].

Figure 2. Wear due to abrasion [8]

Figure 3 indicates that, according to its nature, abrasive wear may be classified in two types, namely, of two or three bodies. The abrasive wear of two bodies is generally used as a machining mechanism, to obtain specific results in a particular surface, whereas the abrasive wear of three bodies is due to the contamination of the interface between two surfaces.

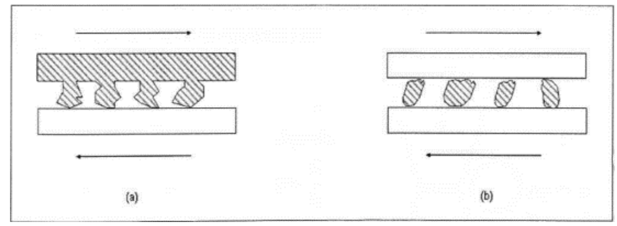

Figure 3. Abrasive wear a) 2 bodies y b) 3 bodies [9]

1.4. Failure analysis in gears

The American Society for Metals has created four subgroups to classify failure modes, namely, wear, surface fatigue, deformations and crack. Table 1 shows the percentages of the common failures, which take into account the parameters that are established due to the abrasive wear of gears, analyzing each failure mode and consequence.

1.5. Matlab

Matlab is a very powerful and adaptable tool for mathematical calculation, with graphical capabilities that improve the data presentation experience. As a consequence of these features, it has become popular as an option for making calculations in science and research [10].

Table 1. Causes of failures in gears [11]

Cause of failure	Percentage %
Related to the service (Total)	74,7
Inappropriate assembly	21,2
Inadequate lubrication	11
Continuous overloads	25
Impact loads	13,9
Bearing failures	0,7
Foreign material	1,4
Equipment operation errors	0,3
Abusive operation	1,2
Related to Thermal Treatment (Total)	16,2
Excessive hardness of the gear body	0,5
Insufficient hardness of the gear body	2
Excessive depth of the coating	1,8
Insufficient depth of the coating	4,8
Inappropriate hardening	5,9
Inappropriate tempering	1
Related to The Design (Total)	6,9
Inappropriate design	2,8
Inappropriate material selection	1,6
Inappropriate treatment specification	2,5
Related to Manufacturing (Total)	1,4
Burns due to grinding	0,7
Marks of tools Marks or notches	0,7
Related to the Material (Total)	0,8
Forging defects	0,1
Steel defects	0,5
Steel mixture or composition errors	0,2

One of the most important Matlab features in the interactive user interface, which enables a fast numerical calculation and an efficient data processing. In addition, it has various functionalities, such as the presentation of graphical tools that enable that user's experience is simpler, pleasant and efficient enough to fulfill all needs, without requiring many software [12].

1.5.1. GUI Tool

A Graphical User Interface (GUI) is a software package within Matlab, that uses a set of preprogrammed images and action boxes to synthetize the need of the user for managing data and tasks.

Figure 4 determines the main functionality of this tool, which is to facilitate the communication between the user and the software, so that it is not necessary to consider programming processes and the language required to modify and create the graphical interface [13].

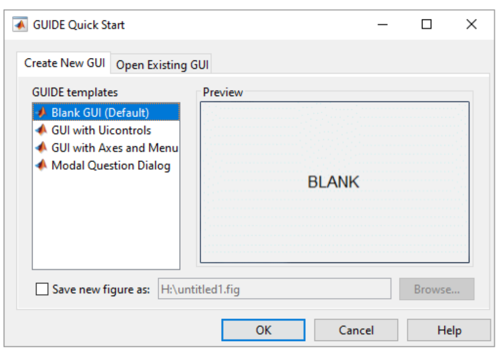

Figure 4. Interface "GuideMatlab"

2. Materials and methods

The aim is to obtain the necessary theoretical-conceptual tools, evaluate the formulas presented in the selected bibliography and adapt them to the circumstances of the problem, and appropriately arrange, interpret and use the data obtained.

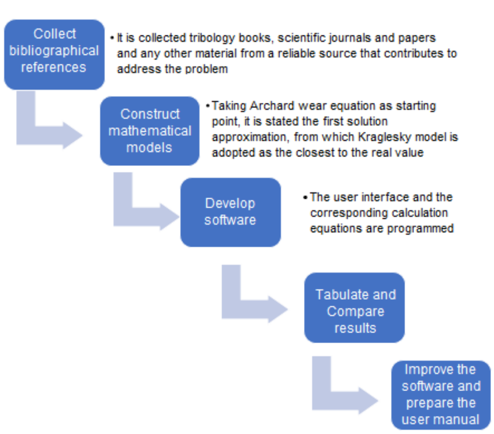

Figure 5. Flow diagram of the methodology [14]

For this purpose, dependent and independent variables are defined and, based on this information, it is formulated a software capable of calculating the abrasive wear rate of gearwheels through a Matlab graphical user tool. The flow diagram of the methodology is illustrated in Figure 5.

2.1. Techniques for collecting information

The techniques used during this research work will focus on two main aspects, as described below.

2.1.1. Documentary-Bibliographic

It has been collected books, certified journals, scientific papers and user manuals about the abrasive wear of gearwheels and the use of Matlab. This information will be the base to discretize and adapt the calculation equations and the mathematical models to the problem stated, and will guide the process of constructing the Matlab graphical tool.

2.1.2. Theoretical and experimental

Once the software has been developed, the validity of equations will be evaluated based on the results obtained, the similarity between the mathematical models and the trends of the wear rate. A value of error will be obtained when the parameters of the equations are varied, and it will be intended to adjust the constants of the equations to get an appropriate fit to real values, that enable to obtain the initial approximations of the design. The data obtained in the real approximations carried out in the operation tests is taken into account [4].

2.2. Fundamentals of gears

Since gears are key for this work, it should be remarked the importance of the fundamental tools and knowledge to determine the geometry, type, materials and manufacturing processes of the most common gears, to correctly do the calculation and dimensioning process.

2.2.1. Terminology

To introduce the analysis and study of gears, it is necessary to define the terminology shown in Figure 6. The INEN 1143 standard about gears states that:

- **Teeth of a gear** Elements that carry out the thrust work, transmit power and have a characteristic profile according to their arrangement.
- **Outer circumference.** Part of the circumference of the gear shape that limits it on its outmost part.
- **Inner circumference.** Part that limits the base of the teeth, also known as root.

- **Primitive circumference.** Circumference formed due to the rotation of the contact points of the gear teeth involved in the process.
- **Addendum.** Perpendicular distance between the pitch circle or primitive circumference and the highest point of the teeth.
- **Helix angle.** Angle formed by the base of the cylinder and the teeth of a helical or screw gear with involute profile.
- **Gear or crown.** It refers to the largest gear in an arrangement of gears.
- **Pinion.** Smallest gear, typically in charge of transmitting movement.
- **Eccentricity.** Is the offset between the common centers of two circumferences.
- **Face width.** Length of the tooth at the plane located at 90 degrees of the gear formation plane.
- **Gear ratio.** Ratio between the greatest and the smallest number of teeth in meshed gears.
- **Module.** Ratio between the pitch circle diameter (in millimeters) and the number of teeth.
- **Pitch.** Distance between a point of a tooth and the same point in the adjacent tooth. It is an indication of the tooth size.
- **Reference line.** Imaginary flat surface tangent to the pitch surfaces of two gears; it is basically the plane that limits the contact points between the gears.
- **Pressure angle.** Angle between the pressure line of the tooth and the flat tangent to the pitch surface. It is basically the direction normal to a gear tooth [15].

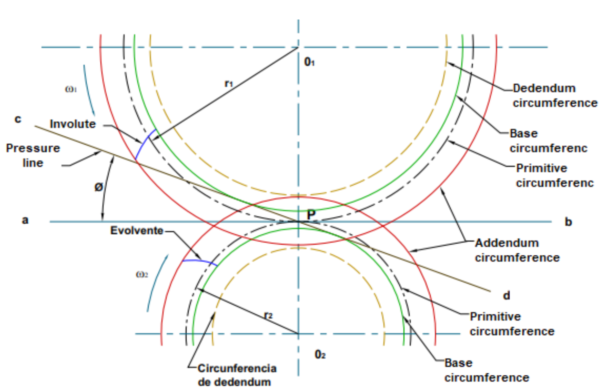

Figure 6. Elements that constitute a gear [16]

2.3. Spur gear

Consider that the total force in a gear is given by Equation (1).

$$F = 19100 \times \frac{power}{Do \times n} \tag{1}$$

Where: F is the total contact force in gears, P is the power of the machine given in KW, Do is the outer diameter and n is the angular speed. The outer diameter is given by Equation (2).

$$Do = z \times m + 2mcos(tan^{-1}(z)) \tag{2}$$

Where z is the number of teeth and m is the gear module.

For the spur gear, the normal force applied in the process is given by Equation (3).

$$Pnormal = Fsen\theta \tag{3}$$

With θ equal to the pressure angle of the gear [17].

2.4. Helical gear

The normal force applied for helical gears is given by Equation (4).

$$Pnormal = F \times sen\phi t \times cos\psi \tag{4}$$

Where: ψ is the helix angle and ϕt is the angle of transverse pressure.

To incorporate time as a variable in the base Archard formula of Equation (5), it is necessary to relate the distance traveled in the abrasion process with the linear displacement of the gear surface when the machine is in operation.

$$L = \frac{Dp \times n \times t}{2} \tag{5}$$

Where n is the angular speed, t is the time variable, and Dp the primitive diameter of the gear, given by Equation (6):

$$Dp = z \times m \tag{6}$$

Where Do is the outer diameter, and z is the number of teeth in the gear.

2.5. Calculation of abrasive wear

According to Equation (7), the volume loss (W) in a piece is directly proportional to the probability (z) that an abrasive particle removes material when it finds a crest of the surface in its trajectory, and to the normal force (N) that acts between the sliding surfaces and the abrasive particles, and is inversely proportional to their hardness measured in the Brinell scale (HB) [18], i.e.

$$W = z \times \frac{N}{HB} \tag{7}$$

Equation (8) shows the abrasive particle that has semispherical shapes with a radius given by the radius of the contact point between the surfaces.

$$W = \frac{k}{3} \times \frac{N}{HB} \tag{8}$$

Where k is the probability of finding an abrasive particle of the contact point between the surfaces, and varies between 10^{-2} and 10^{-7} [19].

The methods of Equation (9) to calculate the wear due to abrasion in various machines elements were tested experimentally and widely by the scientific community, recognizing the nature of wear due to fatigue, and finally, Kraglesky equation will be taken as reference.

$$V = \frac{A \times K}{M} \tag{9}$$

Where (V) is the wear rate measured in [um/h], (A) is the parameter that characterizes the abrasive material, (K) is the characterization of the geometrical conditions of the contact point of the sliding surfaces and (M) depends on the properties of the material of the surface [3].

The base equation to measure the wear rate in spur and helical gears is obtained from these considerations.

- Archard Equation: Wear rate in a spur gear [3] (10).

$$V = K_{process} \frac{\frac{19100 \times power}{Do} \times sen\theta \times \frac{Dp \times n}{2}}{H} \tag{10}$$

- Archard Equation: Wear rate in a helical gear [3] (11).

$$V = K_{process} \frac{\frac{19100 \times power}{Do} \times sen\theta t \times cos\psi \times \frac{Dp \times n}{2}}{H} \tag{11}$$

- Kraglesky abrasive wear rate (12).

$$V = \frac{A \times K}{M} \tag{12}$$

The variables necessary to characterize Kraglesky equation (Equation (12)) are divided in three parameters, the term corresponding to the abrasive particle, where the mechanical properties, size and composition of the abrasive material modify its characteristics according to Equation (13) [20].

- Properties of Kragelsky abrasive particle.

$$A = \epsilon^{2/3} r^{0.5} \vartheta^{2.5} \tag{13}$$

In this same context, factor (M) of Equation (14) is related to the mechanical properties of the base material, and is directly proportional to the hardness and the stretching percentage of the material under analysis, taking the abrasive data from the values in Table 2 [20].

Table 2. Size of the types of abrasive particles

Granulometry of the soil	**Particle size range (mm)**
Brick	600 >
Stone	250 - 600
Gravel	75 - 250
Coarse-grained sand	0,5 - 1
Medium-grained sand	0,25 - 0,5
Fine-grained sand	0,05 - 0,25
Clay	< 0,002

- Kragelsky parameter of mechanical properties of the material (14).

$$M = \epsilon_0^t HB_1^{1.5} HB_2 \tag{14}$$

Where ϵ_0^t corresponds to the stretching percentage of the material before the crack, t is a nondimensional parameter associated to the contact between bodies, and HB represents the hardness of the materials that constitute the tribological pair of the process, measured in Brinell scale. Finally, factor (K) involves all the geometrical conditions that impact the variation of the contact between the surfaces, such as the type and size of pieces, lubrication conditions and distribution of the contact forces between the elements of the tribological pair. For this reason, Equations (15) and (16) are established for spur and helical gears, respectively [20].

- Spur gear

$$K = (m \times (z1 + z2) sin\vartheta)^{0.5} \times 0.106 \times n \tag{15}$$

- Helical gear

$$K = \left(\frac{m \times (z1 + z2) sin\vartheta}{cos\psi(1 - cos\vartheta^2 \times sin\psi^2)} \right)^{0.5} \times 0.106 \times n \quad (16)$$

Therefore, according to Kraglesky, the calculation of abrasive wear rate is summarized as follows (17) and (18).

- Kraglesky Equation: Wear rate spur gear

$$V = 576 \frac{\left(\epsilon^{2/3} r^{0.5} \vartheta^{2.5}\right)}{\varepsilon_0^t HB_1^{1.5} HB_2} (m \times (z1 + z2) sin\psi)^{0.5} \times 0.106n \quad (17)$$

- Kraglesky Equation: Wear rate helical gear

$$V = 576 \frac{\left(\epsilon^{2/3} r^{0.5} \vartheta^{2.5}\right)}{\varepsilon_0^t HB_1^{1.5} HB_2} \left(\frac{(m \times (z1 + z2) sin\psi)^{0.5}}{cos\psi(1 - cos\vartheta^2 \times sin\psi^2)} \right)^{0.5} \times 0.106n \quad (18)$$

2.6. Designation of variables for calculating the abrasive wear rate

Archard equation was used as a first approximation for calculating the abrasive wear rate in gearwheels. On the other hand, the mathematical model formulated by Kragelsky, given by Equation 3, was used to determine the result closest to the real value. Afterwards, both methods were programmed in a beta version of the software designed with the Matlab graphical tool. Then, results were generated to test the calculation application, aspect issues were improved and the software mas made as friendly as possible for the user.

At last, the final version of the application was developed and compiled to make it independent of Matlab. Then, the variables and the parameters of the equations are changed, and the data obtained is used to make plots in Excel. The final step involved preparing a user manual and presenting the results.

2.7. Declaration of variables

Based on the data required by the equations and to clearly state the names of the variables used in the programming process, this section presents.

ϑ (theta) = pressure angle
ψ (psi) = helix angle
$z1$ = number of teeth of gear 1
$z2$ = number of teeth of gear 2
m = module of the gears
P = power of the machine
$H1$ = hardness of material 1 (Brinell)
$H2$ = hardness of material 2 (Brinell)
$Karch$ = process constant, Archard equation
n = angular speed
rg = average grain size of the abrasive particle
cv = concentration in volume of the abrasive particle
$Eo1$ = stretching percentage of the material of gear
$Eo2$ = stretching percentage of the material of gear

3. Results and discussion

3.1. Results

3.1.1. App version

When entering the app (see Figure 7), it should be selected the gear type and the known parameters. The interface enables a clear view of the requirements, such as number of teeth of the gear, gear diameter, pressure angle, helix angle, gear type, power required, angular speed, among the most important factors for recording the data necessary for the calculation.

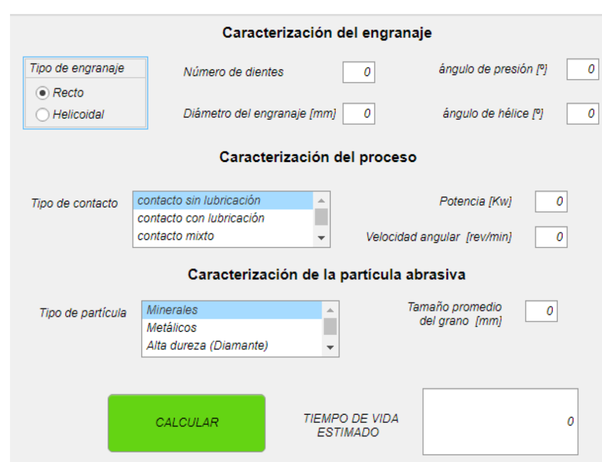

Figure 7. App for calculating the wear rate

Afterwards, the necessary inputs and outputs of the system are evaluated for each equation, as shown in Figure 8. Then, a second test version of the model was generated taking into account details that improve user's experience, such as a help button, a table for unit conversion, as well as other factors to avoid unnecessary formulation errors. In other words, it is sought to avoid repetitive data, or to minimize the data that should be entered when the values necessary can be calculated from the ones already obtained.

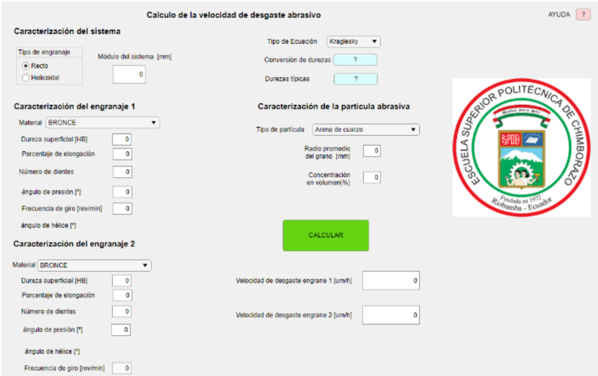

Figure 8. Version 2 of the App for calculating the wear rate

Based on the necessary variables and on the requirements for the correct operation of the software for calculating the abrasive wear rate of gearwheels, using the Archard and Kraglesky methods, Figure 9 shows the final interface developed so that it is user friendly and has a pleasant look.

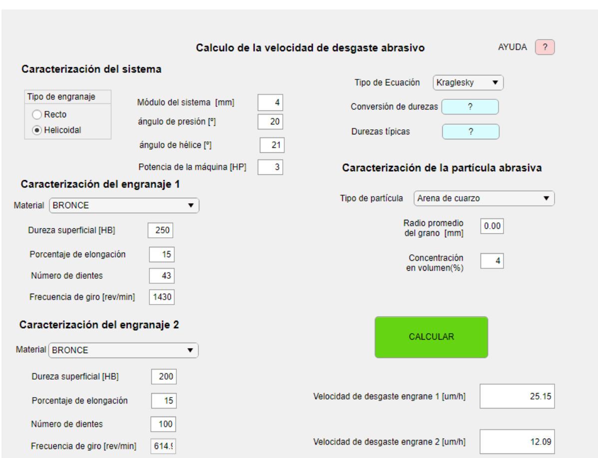

Figure 9. Versión final APP

3.1.2. Collection of final data

At this point, it is sought a comparison of the mathematical models to point out their differences, and compare the results for the cases of plots with spur and helical gears. In this manner, it is possible to see a trend in the models to assure that there are no data with significant errors, and verify the validity of the model in each case. Hence, isolated cases are limited, and it is understood when and under which conditions it is relevant to use each of the models stated.

For this purpose, two simple analyses are performed. In the first one, which is shown in Table 3, the number of teeth of one of the gears is changed, and it is intended to calculate the wear for both spur and helical gears, for each of the gearwheels involved in the process.

The hardness and the stretching percent of the base material of both wheels are kept constant at 250 HB and 18%, respectively; it is also kept constant at 1430 rpm the frequency of gearwheel one. The module of the system is set at 4, and the pressure angles at 20 and 21 degrees, respectively.

The average power of the machine to be evaluated was set at 200 HP for Archard equation. Regarding the particle characterization for Kraglesky equation, it was used quartz sand with an average grain size of 0.05 mm and 4% of concentration in volume in the surrounding medium [20].

Table 3. Wear rate with variable number of teeth [14]

Number of teeth		Spur gear				Helical gear			
		Wear speed 1		Wear speed 2		Wear speed 1		Wear speed 2	
z1	z2	Archard	Kraglesky	Archard	Kraglesky	Archard	Kraglesky	Archard	Kraglesky
43	10	17,42	15,03	73,54	64,63	18,66	16,52	78,77	71,03
43	12	17,42	15,31	61,65	54,86	18,66	16,83	66,03	60,3
43	15	17,42	15,72	49,56	45,07	18,66	17,28	53,09	49,54
43	20	17,42	16,39	37,31	35,23	18,66	18,01	39,97	38,72
43	25	17,42	17,02	29,91	29,28	18,66	18,71	32,03	32,18
43	43	17,42	19,14	18,66	19,14	18,66	21,04	18,66	21,04
43	50	17,42	19,91	14,99	17,12	18,66	21,88	16,05	18,82
43	60	17,42	20,95	12,49	15,02	18,66	23,03	13,38	16,5

The wear values for spur gears under the aforementioned variable specifications are determined in Figures 10 and 11; these values are changed to obtain the data plotted in Excel, to be able to appropriately manage the parameters of the approximations, taking into account that Kraglesky mathematical equation is the appropriate one for analyzing the closeness to the values under data variability.

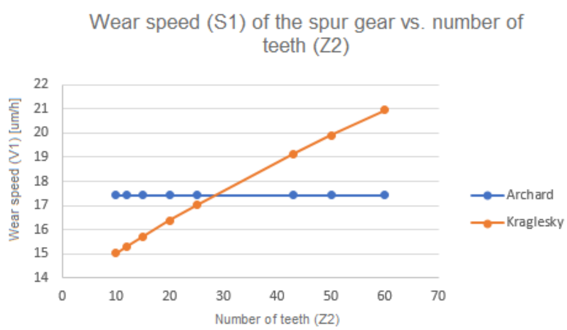

Figure 10. Wear rate 1 of spur gears with Z2

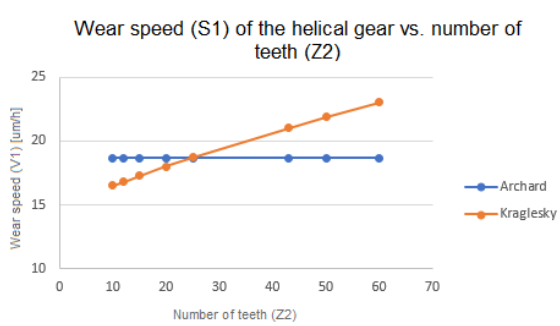

Figure 11. Wear rate 1 of helical gears with Z2

The results of both Archard and Kraglesky models are plotted in an Excel sheet for spur gears (Figure 12) and helical gears (Figure 13).

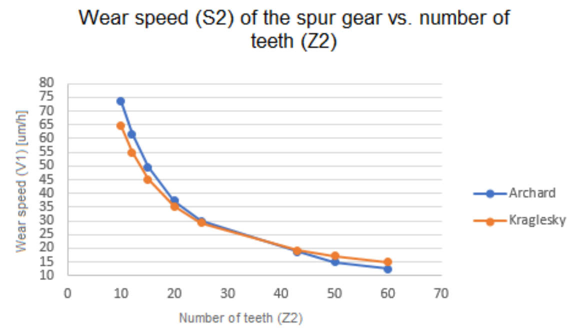

Figure 12. Wear rate 2 of spur gears with Z2

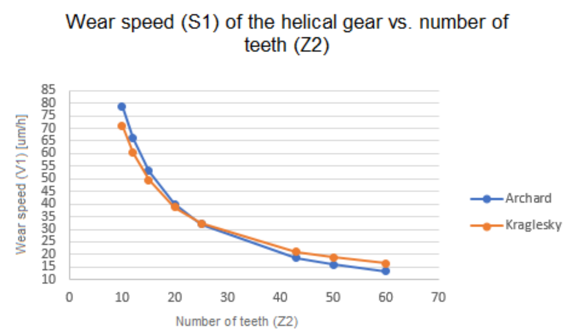

Figure 13. Wear rate 2 of helical gears with Z2

The previously stated data will be considered for the second case of analysis, with the difference that the number of teeth of the gears involved will be kept constant at 43, and the surface hardness of the second gear will be varied. Similarly, it will be calculated the wear rate for each gear, either spur or helical. In this manner, the data given by the final version of the App will be presented in detail, where another comparison is made according to the resistance HB2 (Table 4).

Table 4. Velocidad de desgaste con dientes variables [14]

Material hardness		Spur gear				Helical gear			
		Wear speed 1		Wear speed 2		Wear speed 1		Wear speed 2	
HB1	HB2	Archa	Kragle	Archa	Kragle	Archa	Kragle	Archa	Kragle
250	150	17,42	31,91	29,04	41,91	18,66	35,07	31,11	45,28
250	170	17,42	28,15	26,62	34,14	18,66	30,95	27,45	37,53
250	190	17,42	25,19	22,93	28,9	18,66	27,69	24,56	31,76
250	220	17,42	21,76	19,8	23,19	18,66	23,91	21,21	25,49
250	250	17,42	19,14	17,42	19,14	18,66	21,04	18,66	21,04
250	280	17,42	17,09	15,56	16,15	18,66	18,79	16,66	17,75
250	300	17,42	15,95	14,52	14,56	18,66	17,54	15,55	16,01
250	350	17,42	13,67	12,45	11,56	18,66	15,03	13,33	12,7

3.2. Discussion

Considering the real approximations performed, it is taken into account the scheme based on vibrations proposed in [4] for updating a wear prediction model. First, it is developed a dynamic model of a system of spur gears to generate realistic vibrations, which enables performing a quantitative study of the effects of wear on the surface of the gear teeth.

The sliding speed and the contact forces are used in combination with Archard wear model, to calculate the depth at each contact point of the mesh. The profile of the teeth of the worn-out gear is fed back in the dynamic model as a new geometrical transmission error, which represents the offset of the profile from an ideal involute curve; hence, it is zero for perfect gears.

Since the wear coefficient of the model is not constant during the wear process (and difficult to estimate initially in any case), the vibrations measured are compared with the ones generated by the model, to update the coefficient when offsets in the predictions are detected.

3.2.1. Data for the calculation

The following data obtained from a study are considered to determine the evaluation process:

Acero dulce AISI 1045

Module: 4

Pressure angle: 20

Power: 4 kW (5,3641 HP)

Hardness: 163 HB

Stretching percentage: 16

Number of teeth: pinion 19 gear 52

Frequency: 6000 rpm

Grain average: 0.05

Concentration of the volume: 1.85 [4]

Figure 14 shows the values determined in the paper under analysis, which specifies the values of the experimental model and Archard model, calculated by the authors; the data obtained from this paper is used for the comparative analysis between the experimental process and the calculations carried out by the Matlab App developed in this work.

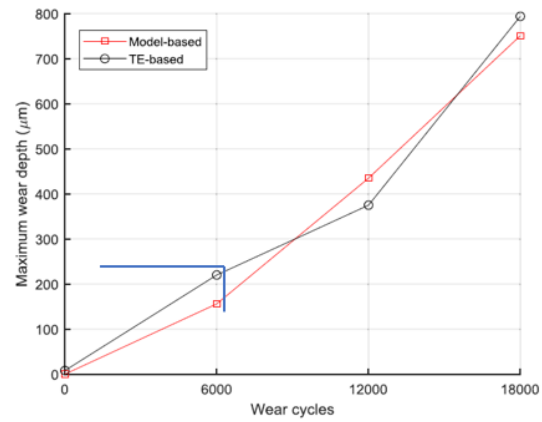

Figure 14. Results for the comparison of the maximum wear depth: experiment and model [4]

3.2.2. Evaluation of the experimental model from the paper and Kraglesky model in the Matlab App.

The error percentage is calculated in this subsection, to verify the coincidence between the data of the application from Kragelsky equation and the data of the experimental model, to plot the analysis model of the abrasive wear prediction with the use of vibrations.

A primary analysis is performed between the mathematical model (blue line) and the experimental model (orange dots). Figure 15 shows the data obtained from the calculation with Kragelsky mathematical model (gray line), used in this research work, and Archard equation (yellow line).

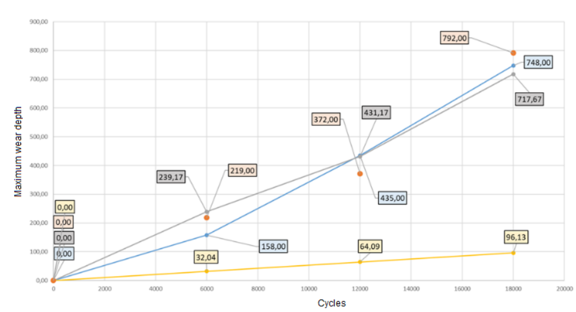

Figure 15. Calculation of proximities between the values given by the App and the experimental paper [14]

Based on the above, it will be verified if the data calculated through the Matlab application are within the permissible ranges to consider them as appropriate for comparing the two situations, namely, Kraglesky equation and the data obtained from the experiment.

The mean quadratic error is considered for the statistical calculation, since it enables to compare the difference between the estimator and what is being estimated. This function is a risk evaluator corresponding to the expected value of the quadratic loss. The difference is due to the randomness or because the estimator does not take into account the information that could produce a more precise estimation (ecuation (19)).

$$ECM = \frac{1}{n}\sum_{t=1}^{n}(Ye - Yc)^2 \quad (19)$$

The mean square error (MSE) given by Equation (19) is used to calculate the difference of the randomness process to produce a more precise estimation. The data chosen corresponds to the data that will be used to determine the expected loss.

Table 5 shows the values calculated, which reflect the effectiveness of the proximity with the data of the experiment. In particular, Krageslsky mathematical model is effective for predicting the wear index, with a 48.56 %, which indicates that the data calculated are within the range of the experimental values of wear. It is convenient to specify that the mathematical model of the paper yields an approximation of 49.06 % with respect to the experimental values.

Table 5. Data of the mean square error [14]

	Experiment vs Kragelsky (APP)	**Experiment vs model in paper**
	0,00	0,00
	406,69	3721,00
	3500,69	3969,00
	5525,44	1936,00
Total sum	9432,83	9626,00
1/n	2358,21	2406,50
Calculated value	48,56%	49,06

On the other hand, Archard equation is analyzed for the identification of the values calculated; Figure 15 depicts that the data show a significant offset, with an error close to 90 %.

It is important to remark that the analysis shows that Kragelsky equation is more effective than Archard equation, since it has data that encompasses the abrasive size and percentage; for this reason, it is feasible for determining the wear index in mechanical elements.

4. Conclusions

Archard and Kraglesky mathematical models have been selected, and it has been correctly identified the necessary variables and the usage of each of these models for calculating the abrasive wear rate. Based on the analysis with the paper under study and the analysis of the quadratic error between the experimental values and the values obtained with the equations, it was determined that Kragelsky model shows a better performance, since it yields a quadratic error of 48.56 % with respect to the experimental data, compared to a quadratic error of 49.06 % given by the mathematical model of the paper. The limitations of this research work are that the data necessary for the calculations should be obtained after the element has been designed and it is also known the construction material.

Regarding the graphical tool created using the Matlab user graphical application (GUIDE), it fulfilled the requirements to calculate the abrasive wear rate in gearwheels, needing prior knowledge of the variables that the user should enter as well as the data of the type of abrasive to be analyzed. The programming was made according to the mathematical model under study, avoiding unintentional errors, and providing options to help the user to determine the unknown variables.

The results obtained with the software were tabulated and plotted in Excel to compare their trends, as shown in the results section, concluding that the best decision for the design of a gear with specific features can be made after the data has been varied; in addition, the designer has available data that may improve the functionality of the mechanical element designed, taking into account the abrasive found in the environment.

Based on the verification of the data obtained from the app and the experiment, besides the explanation provided, it is concluded that the software developed in this research work enables to calculate the abrasive wear rate on spur and helical gearwheels with involute profile. It also establishes prediction data for decision making regarding the execution of maintenance programs, and the prevention of catastrophic failures that may harm the working area and the personnel in charge of the machinery. The contribution of the software is to become a tool to determine values that enable the mechanical designer to perform a quick evaluation of the construction of a mechanical element, based on variables that may be found in the gear construction process.

Future works

The perfect companion for the software developed is a short-term database that contains experiments with gears and different abrasives, to verify the data and similarly calibrate the constants of the processes for different working conditions under lubrication regime and controlled operation. In addition, to assess the data with statistical tools for improving the error percentage, and that the data calculated with the Matlab app are as close as possible to the data obtained when executing an experimental process.

References

[1] J. C. Vesga Ferreira, G. Granados Acuña, and J. E. Sierra Carrillo, "Optimización de una red

multiservicio sobre un canal PLC bajo MmQoS," *Ingeniería y Desarrollo. Universidad del Norte*, vol. 33, no. 2, pp. 260–280, 2015. [Online]. Available: http://dx.doi.org/10.14482/inde.33.2.6368

[2] P. N. Martínez Henao and N. A. de la Peña, "Estado del arte sobre perfiles de dientes para engranajes," *Prospectiva*, vol. 12, no. 2, pp. 29–40, 2014.

[3] A. Fernández del Rincón and P. García Fernández. (2019) Máquinas y mecanismos. [Online]. Available: https://bit.ly/3UG1X4K

[4] K. Feng, P. Borghesani, W. A. Smith, R. B. Randall, Z. Y. Chin, J. Ren, and Z. Peng, "Vibration-based updating of wear prediction for spur gears," *Wear*, vol. 426-427, pp. 1410–1415, 2019, 22nd International Conference on Wear of Materials. [Online]. Available: https://doi.org/10.1016/j.wear.2019.01.017

[5] I. D. Marinescu, W. B. Rowe, B. Dimitrov, and H. Ohmori, *Tribology of Abrasive Machining Processes*. Elsevier Inc., 2013. [Online]. Available: https://doi.org/10.1016/C2010-0-67070-2

[6] G. W. Stachowiak, *Wear - Materials, Mechanisms and Practice*. John Wiley & Sons, Ltd., 2005.

[7] Robles, *Desgaste*. Universidad Autónoma de Nuevo León, 2012. [Online]. Available: https://bit.ly/3X2BkbV

[8] L. Marciano Vázquez, "Diseño de engranajes rectos y helicoidales asistido por computadoras," Master's thesis, 2000. [Online]. Available: https://bit.ly/3X2OWno

[9] Motorex. (2019) Qué es un engranaje recto. [Online]. Available: https://bit.ly/3EbMX7S

[10] D. O. Barragán Guerrero, *Manual de interfaz gráfica de usuario en Matlab*. Escuela Superior Politécnica del Litoral, 2013. [Online]. Available: https://bit.ly/2UUgwD2

[11] D. S. Romero Lozano and J. A. Giraldo Ramos, *Análisis de falla de un engranaje de la caja de cambios de una motocicleta*. Fundación Universitaria Los Libertadores, 2018. [Online]. Available: https://bit.ly/3E9NtDk

[12] Hardwaresfera. (2020) Definición de software de sistema, de programa y de aplicación, además de la definición de software libre y software propietario. [Online]. Available: https://bit.ly/2UVU644

[13] G. Cid Espinoza, "Programación de interfaz gráfica en app designer MATLAB," 2018. [Online]. Available: https://bit.ly/3O6EXcK

[14] J. M. Mena Chavarrea, "Cálculo de la velocidad de desgaste abrasivo en engranajes de dientes rectos y helicoidales con perfil evolvente, utilizando una GUI de Matlab," 2021.

[15] INEN, "Engranajes. terminología para negranajes helicoidales cruzados 90°," 1983. [Online]. Available: https://bit.ly/3g3TeKT

[16] J. L. M. Rodríguez and J. A. Velázquez Pérez, "Cálculo de engranajes plásticos," *Ingeniería Mecánica*, vol. 10, no. 3, pp. 25–32, 2007. [Online]. Available: https://bit.ly/3UCqO9v

[17] H. W. Gamboa Mariño and D. R. Vistin Guilcapi, "Análisis tensional en transmisiones de engranajes rectos mediante el método de los elementos finitos," 2006. [Online]. Available: https://bit.ly/3UDsVtV

[18] Masterhacks. (2017) Fundamentos de engranajes. [Online]. Available: https://bit.ly/3UJtpyD

[19] Engranabreider. (2016) Engranajes. [Online]. Available: https://bit.ly/3X7SCEr

[20] I. V. Kragelsky, M. N. Dobychin, and V. S. Kombalov, *Friction and WearCalculation Methods*. Elsevier Ltd., 1982. [Online]. Available: https://doi.org/10.1016/C2013-0-03333-6