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Empirical Estimation of the Prestressed of a V-Belt Through the Slip of the Pulleys

Estimación empírica del pretensado de una banda trapezoidal mediante el deslizamiento de las poleas

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Abstract

Inspecting the pre-tension of belts is an essential preventive maintenance activity, which requires that the machine is powered off to be carried out. This generates an economic impact of lesser or greater degree depending on the operational context of each machine. The objective of this experimental investigation is to determine a mathematical model, for calculating the pre-tension of v-belts of classic profile and high performance as a function of the slip. To achieve this objective, a test module was built to establish the difference between the theoretical and real rotation frequencies of the driven pulley, as the pre-tension of the belt was increased. Then, an inverse exponential function was adjusted to the data obtained, resulting in two equations for v-belts of classic profile and of high throughput, respectively; these equations were validated using Pearson's r correlation coefficient. The proposed mathematical model can be used to minimize the economic impact of checking the pre-tension of the belts, since it allows carrying out this activity with the machine operating on full load, requiring only the measurement of the rotation frequencies of the pulleys.

Palabras clave: v-belt, slip, pulley, prestressed of the belt.

Resumen

La inspección de la pretensión de las bandas es una actividad de mantenimiento preventivo imprescindible; que, para poder ser realizada, requiere que la máquina esté apagada, generando un impacto económico en menor o mayor grado dependiendo del contexto operacional de cada máquina. El objetivo de esta investigación experimental es determinar un modelo matemático para el cálculo del pretensado de las bandas trapezoidales de perfil clásico y de alto rendimiento en función del deslizamiento; para lo cual se construyó un módulo de prueba, en el que se estableció la diferencia de las frecuencias de rotación entre la teórica y la real de la polea conducida, a medida que se incrementó el pretensado de la banda. Los datos arrojados se ajustaron a una función exponencial inversa, dando como resultado dos ecuaciones, una para las correas trapezoidales de perfil clásico y otra para las de alto rendimiento. La validación de estas ecuaciones se realizó mediante el coeficiente de correlación r de Pearson. Con la utilización del modelo matemático propuesto, se podrá minimizar el impacto económico de las actividades preventivas de revisión del pretensado de las bandas; puesto que, estas ecuaciones posibilitan la realización de esta actividad con la máquina encendida y a plena carga, requiriendo para ello únicamente la medición de las frecuencias de rotación de las poleas.

Keywords: banda trapezoidal, deslizamiento, polea, pretensión de bandas.

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1. Introduction

Nowadays v-belts are extensively used to transmit power in industrial machines and vehicular applications, which do not require a precise velocity and where moderate power levels are sufficient. The cost of these belts is smaller compared to transmission by chains or gears, and its operation is relatively noiseless [1, 2]. In addition, they have the characteristic of absorbing shock loads and reducing the effects of vibration [3].

In the texts about the design of machine elements, the selection of v-belts considers that the power to be transmitted should be smaller than the power that the belt, or set of belts, are capable of sustaining. This process also includes determining the initial tension or prestressed to guarantee power transmission without slipping [4–6].

However, depending on the operating conditions, in these transmission systems a slip between the belt and the pulley always exist, thus the transmission ratio is not constant [7, 8].

The slip mechanism of flat and v-belts has been extensively studied, both analytically and experimentally. Reynolds [9] demonstrated that the velocity loss is due to the elastic slip of the belt.

Gerbert [10–12] developed two theories mainly related to bending and deflection due to shearing (classical creep theory); however, the slip is much greater than what is forecasted by this theory. Afterwards, he developed a new unified theory that fitted better the experimental observations, but fails at low tension.

Belofsky [13] proposed a model that takes into account the elasticity of the belt, the resistance to bending and the variation of the friction force along the contact arch, based on a linear slip regime. This work describes an experimental method for determining μ at any point of the contact arch of the belt.

Childs and Cowburn [14] analyzed the belt bending resistance, the radial adjustment, the distortion due to the diameter of the pulley and the prestressed of the belt, to describe the velocity and torsion loss in v-belts transmission.

Chen and Shieh [15] obtained the velocity loss in a flat belt transmission system, using a 3D finite elements model. Balta, Sonmez and Cengiz [16] analyzed a multi-ribbed belt transmission system, and determined that a smaller the pulley size results in a greater belt slip, and that below a certain level of the prestressed of the belt the slip values increase.

In the consulted bibliography it was observed that most of the studies in flexible transmission systems focus on the power loss, and for this purpose the factors that influence the slip and the effects of belt prestressed are analyzed.

The values of the variables in the proposed formulations can be obtained experimentally. However, according to the authors, such values should be determined for every combination of velocity, diameters, distance between centers and pulley materials [17, 18], thus creating multiple possibilities in which it will be very difficult to include particular transmission systems that have been already built, and that take part in an industrial environment where experimental tests cannot be carried out.

Once the belts have been assembled in the transmission system and start operating, they undergo a considerable lengthening in a short period of time, thus causing the reduction of the initial tension; due to this, belt manufacturers recommend doing a prestressed in a period of time not longer than 24 hours. In subsequent days, this lengthening will appear after a much longer undefined period of time [19]. Therefore, it is required to plan a preventive activity at fixed intervals between three and six months, to verify and, is necessary, correct the prestressed [20].

There are methods with and without contact to measure the prestressed belt, which requires the machine is turned off. This requirement does not assume any problem when the machine has an operation regime of five days a week, because any preventive activity is reserved to be carried out in resting days, where the production is not interrupted, but the overtime should indeed be assumed.

In the case of machines with an operating regime of 24 hours a day, 7 days a week, the preventive maintenance activities will cause a system operational downtime and, therefore, a cost associated with non-production [21]; thus, the intervention time must be the minimum possible.

The objective of this work is finding a mathematical model based on experimentation, which may estimate the prestressed belt from direct measurement, with laser tachometers, of the velocities of the driving and driven pulleys, without requiring to turn off the machine nor requiring parameters which are difficult to obtain. Therefore, this will facilitate the work of maintenance technicians.

The main contribution is minimizing the economic impact of the inspection of prestressed belts, because with the aid of the proposed mathematical model, this activity can be conducted without interrupting the normal operation of the machine.

For carrying out this study, a test module was utilized in which the prestressed of the belt can be progressively modified while registering the variation of the produced slip. Then, a mathematical function can be fitted to describe the existing relationship between slip and prestressed.

2. Materials and methods

2.1. Experimental configuration

The configuration is shown in Figure 1 and outlined in Figure 2 it was constructed to investigate the effects of the belt tension on the velocity loss of the driven pulley, in flexible transmission systems with v-belts of classic and high performance profile. The components of the system are firmly supported on an AISI 1020 steel base, with a thickness of 10 mm. In this system, the driving pulley with diameter of 125 mm, rotates due to the mechanical energy supplied by a 373 W asynchronous squirrel cage electric motor, with four magnetic poles; thus, the frequency of rotation of the magnetic field is 30 Hz.

Figure 1. Configuration of the test module.

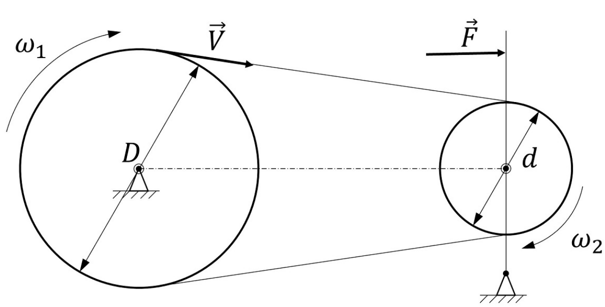

Figure 2. Outline of the belt transmission system.

The driven pulley with a diameter of 70 mm, rotates mounted on an alternator of 12 V, 35 A and a frequency of rotation of 50 Hz. A resistive load is connected to the alternator such that the electric motor supplies 80% of its nominal power.

Considering that the efficiency of the motor is 78.2%, the resistance connected to the alternator was set to the value in which the electric power supplied in the motor was:

$$P = \frac{373W \times 0,80}{0,782} \approx 382W \tag{1}$$

In order to generate tension in the belt, the alternator was attached to the base plate my means of a bolt which lets it turn around its axis, while a hardener was placed in the upper part. A digital weighing scale was inserted to work as a load cell, i.e. to measure the tension in the belt as the hardener is tighten.

This hardener is kept at a constant height, and is attached to the base plate by means of two parallel tension bars of 5 mm thickness. As can be seen in Figure 3, these bars reach a maximum static deformation of 0.9418 mm when subjected to a force of 500 N, the maximum value that can be measured with the scale. This deformation is not considered a value that can influence on the measurements during the tests, thus guaranteeing that it works correctly.

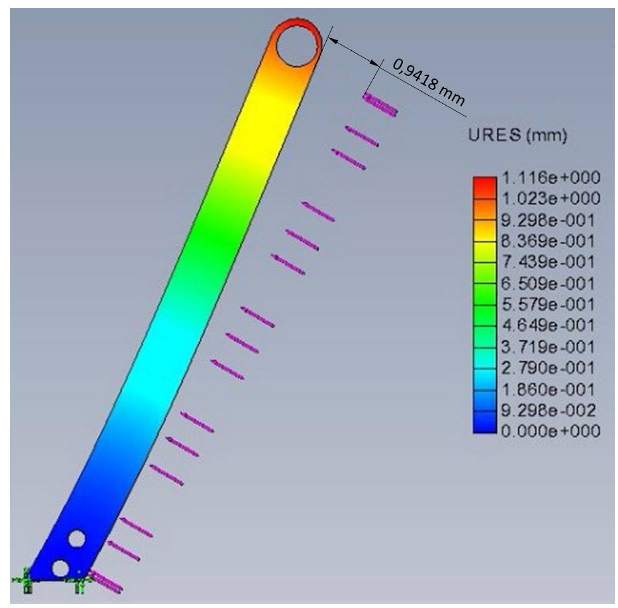

Figure 3. Maximum deformation of the parallel tension bars, obtained using the finite element method in the software Solidworks, version 2016.

Two photo tachometers are used to measure the rotation frequency of the pulleys. Such photo tachometers directly point to 10 mm x 20 pieces of reflective tape stuck away from the center on the outer side of the pulleys.

The tested v-belts included the classic profile A-23 and the high performance SPA-23, both with a length of 584.2 mm and manufactured by Dongil.

2.2. Calculation of slip

The slip produced in the belt transmission system is the ratio of the difference between the theoretical and real (measured) angular velocities of the driven pulley, to the theoretical angular velocity. Hence, this can be mathematically expressed as:

$$Dz = \frac{\omega_{t2} - \omega_2}{\omega_{t2}} \tag{2}$$

where Dz is the slip, ω_{t2} is the theoretical angular velocity of the driven pulley calculated using the equations that describe the uniform circular motion, and ω_2 is the real (measured) angular velocity of the pulley.

Considering that the angular velocity can be expressed in terms of the frequency as:

$$\omega = 2 \times \pi \times f \tag{3}$$

and that, for the uniform circular motion, the theoretical angular velocity of the driven pulley is given by:

$$\omega_{t2} = \omega_1 \times \frac{D}{d} \tag{4}$$

where ω_1 is the real angular velocity of the driving pulley and D and d are, respectively, the diameters of the driving (bigger) and driven (smaller) pulleys given in meters (m), equation (2) can be equivalently expressed as

$$Dz = 1 - \frac{f_2 \times d}{f_1 \times D} \tag{5}$$

where f_1 and f_2 are, respectively, the real rotation frequencies of the driving and driven pulleys, given in Hertz (Hz) or revolutions per minute (rpm).

2.3. Calculation of the belt tension

In order to calculate the belt tension from the scale measurements, a static analysis of the module tensioning system is carried out. Figure 4 illustrates the configuration of such tensioning system, while Figures 4 y 5 shows its equilibrium diagram.

Since the force measured by the scale acts on point H (Figure 4) and the alternator is centered in G, a summation moment in the latter results in:

$$\begin{gathered}\overline{GA} \times \vec{T_2} \times sen(\beta - \varphi) + \overline{GB} \times W \times cos(\beta) + \\ + \overline{GC} \times \vec{t_1} \times sen(\beta + \varphi) - \overline{GH} \times \vec{F} \times sen(\beta) = 0\end{gathered} \tag{6}$$

If the machine is at rest (off, without any rotational movement), the tensions T_1 and T_2 on the belt are equal. Substituting the values of distance (Table 1) yields:

$$T = \frac{3 \times (2F \times sen(\beta) - W \times cos(\beta))}{2 \times (2 \times sen(\beta + \varphi) + sen(\beta - \varphi))} \tag{7}$$

In this rest condition, the tension T calculated using equation (7) is known as prestressed. At this point it should be clarified that once the pulleys start rotating, tensions 1 and 2 become different [3]. This case has not been considered, since the belt prestressed in maintenance operations is carried with the machine at rest (off, i.e. with no motion).

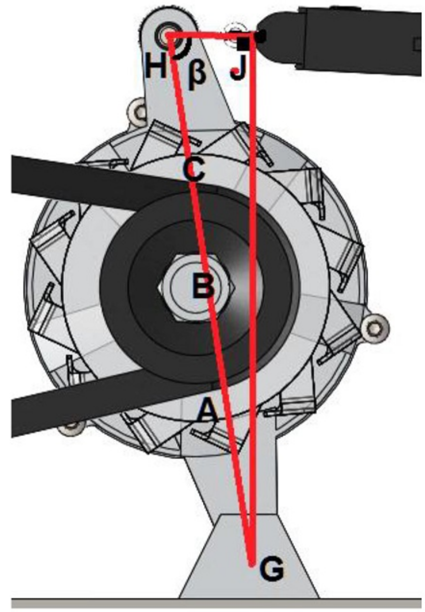

Figure 4. Tension system of the module band.

Table 1. Distances in Figure 4, given in mm

$\overline{GA}$	$\overline{GB}$	$\overline{GC}$	$\overline{GH}$
60	90	120	180

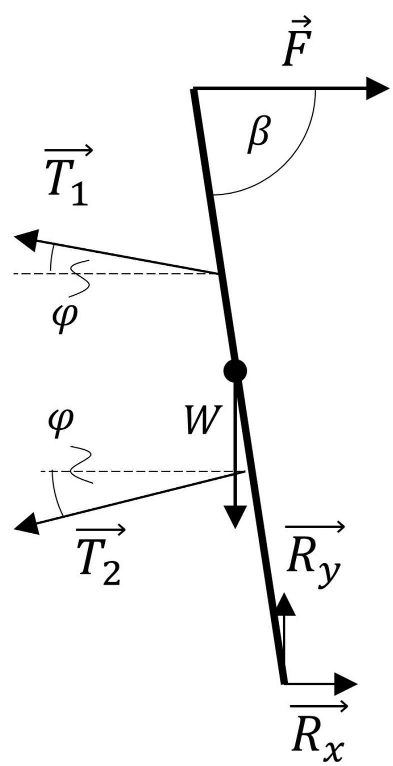

Figure 5. Equilibrium diagram of the tensioning system of the belt in the module.

3. Results and discussion

The results are shown in Tables 2 and 3 for v-belts of classic profile and high performance, respectively, were obtained using the average of five measurements of the pulleys rotation frequencies, and the applied force by the tensioning system at increasing intervals of 50 N. In such tables, the tension T and the slip Dz were calculated by means of equations (5) and (7), respectively.

Table 2. Slip and tension of the v-belt of classic profile

F (N)	T (N)	$f1$ (Hz)	$f2$ (Hz)	Dz
49,05	39,7	29,17	50,43	3,17
98,1	88,22	29,08	50,45	2,86
147,15	136,51	29	50,47	2,55
196,2	185,14	28,92	50,48	2,23
245,25	233,47	28,83	50,5	1,92
294,3	281,79	28,77	50,48	1,72
343,35	330,12	28,63	50,27	1,69
392,4	378,45	28,5	50,05	1,66
441,45	426,77	28,37	49,83	1,62
490,5	475,1	28,23	49,62	1,59

Table 3. Slip and tension of the v-belt of high performance

F (N)	T (N)	$f1$ (Hz)	$f2$ (Hz)	Dz
49,05	39,7	29,3	51,43	1,7
98,1	88,22	29,22	51,38	1,51
147,15	136,51	29,15	51,33	1,38
196,2	185,14	29,08	51,28	1,25
245,25	233,47	29,02	51,25	1,09
294,3	281,79	28,95	51,18	0,99
343,35	330,12	28,92	51,13	0,98
392,4	378,45	28,85	51,03	0,94
441,45	426,77	28,78	50,93	0,91
490,5	475,1	28,72	50,83	0,87

On the other hand, Table 4 indicates the tension of the v-belts of classic profile and of high performance, and the calculated theoretically according to design books, by means of the equation [3].

$$T = \frac{280159.39 \times Hd \times (e^{f\varphi}+1)}{n \times d \times (e^{f\varphi}-1)} \tag{8}$$

where, Hd is the maximum transmitted mechanical power in horsepower (hp), calculated as the product of the measured electric consumption of the motor and the efficiency (0.782 in this case) with an increase of 20% due to the service factor, n is the rotation frequency of the driven pulley in revolutions per minute (rpm), d is the diameter of the same pulley in inches, f is the friction coefficient for v-belts (which is equal to 0,5123 [3]), and φ is the contact angle given by.

$$\varphi = \pi - 2 \times sen^{-1}\frac{D-d}{2 \times C} \tag{9}$$

where D and d are given in mm, and C is the distance between the centers of both pulleys, also in mm.

Table 4. Theoretical and experimentally induced tensions in the v-belts of classic profile and of high performance

***T* teórica de the v-belt of classic profile (N)**	**Theoretical *T* the v-belt of high performance (N)**	**Theoretical *T* of induced *T* (N)**
31,1	27,11	39,7
32,82	28,83	88,22
34,52	30,55	136,51
36,23	31,42	185,14
38,8	33,13	233,47
41,39	34,87	281,79
44,16	36,6	330,12
45,21	38,37	378,45
48,02	40,15	426,77
49,1	41,93	475,1

It can be seen in Table 4 that the theoretical tensions are smaller than the experimentally induced; hence, no slip should exist. Nevertheless, Tables 2 and 3 indicate that it indeed existed, confirming the observations on [7, 8]. Therefore, this research is applicable to any design condition of this type of systems.

Figure 6 is a plot of the slip values as a function of the prestressed for both classic profile and high performance v-belts, from data in Tables 2 and 3.

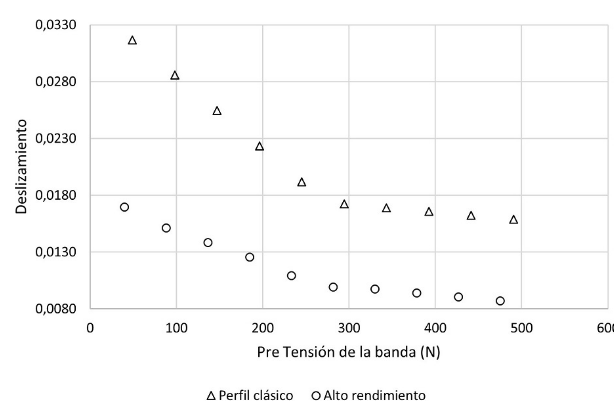

Figure 6. Slip as a function of prestressed for the v-belts.

In order to obtain the mathematical model, the following conditions were considered:

The slip is an index whose value is between zero and 1. As the prestressed increases, the slip decreases toward zero, but it can never take negative values. This means that as the prestressed T goes to infinity, the slip goes to zero, i.e.

$$\lim_{T\to\infty} Dz(T) = 0 \tag{10}$$

If there is not prestressed (it is equal to zero) there is not power transfer between the driving and driven pulleys, thus the latter will not move at all. In this case the slip takes it maximum value of one, which means that the belt totally slips on the driven pulley without transmitting any motion, i.e.

$$Dz(T=0)=1 \tag{11}$$

These conditions are satisfied by inverse exponential models. In particular, a distribution of the type:

$$Dz=e^{-(\alpha T)^{\beta}} \tag{12}$$

was chosen due to its flexibility, where α y β are, respectively, the scale and shape parameters of the model.

In order to obtain the parameters of the model using least squares, it is suggested to linearize Equation (12) through the application of logarithms, to obtain

$$ln\left(ln\left(\frac{1}{Dz}\right)\right)=\beta ln(T)+\beta ln(\alpha) \tag{13}$$

which is used to generate the plot of Figure 7. These data points fit straight lines, with Pearson correlation coefficients r of 0.9767 and 0.9809 for the classic profile and high performance v-belts.

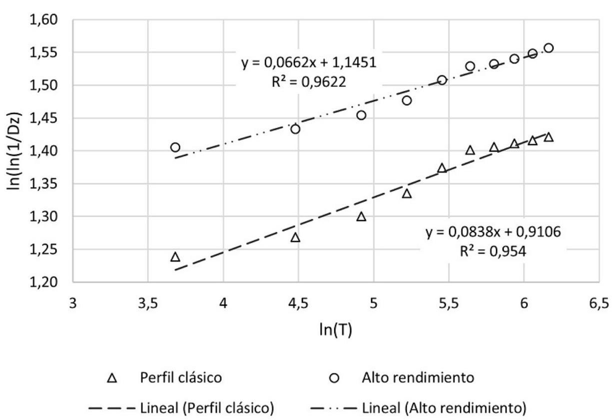

Figure 7. Linearization of the slip curve as a function of the prestressed of the belts.

Then, Table 5 shows the parameters of the linearized models (13) identified using least squares, which result in the mathematical models

Table 5. Parameters of the linearized equations of slip as a function of belt tension

Tipe of band	Slope	Intersection	α	β
Classic profile	0,0838	0,9106	$5{,}27E^{+04}$	0,0838
High performance	0,0662	1,1451	$3{,}19E^{+07}$	0,0663

$$Dz=e^{-(5,27E^{+04}T)^{0,0838}} \tag{14}$$

$$Dz=e^{-(3,19E^{+07}T)^{0,0663}} \tag{15}$$

In order to validate the fit of equations (14) and (15) to the experimental data, the following hypotheses are stated:

$\boldsymbol{H_0}$: The slip is not an inverse exponential function of the prestressed of the belts.

$\boldsymbol{H_1}$: The slip is an inverse exponential function of the prestressed of the belts.

It can be seen that the obtained Pearson correlation coefficients r, 0.9767 for the v-belt of classic profile and 0.9809 for the v-belt of high performance, are both greater than 0.765 for a set of ten data. Therefore, the null hypothesis is rejected, and the alternative hypothesis is accepted which states that the slip is an inverse exponential function of the prestressed of the belts, with a 99% confidence [22].

Solving equations (14) and (15) for the prestressed T, yields the mathematical models (16) and (17).

$$T=1,91E^{-0.5}\times\left[ln\left(\frac{1}{Dz}\right)\right]^{\frac{1}{0,0838}} \tag{16}$$

$$T=3,13E^{-0.8}\times\left[ln\left(\frac{1}{Dz}\right)\right]^{\frac{1}{0,0663}} \tag{17}$$

which constitute the purpose of this research.

4. Conclusions

As it is observed in Tables 2 and 3, and in Figure 6, the slip produced by the same tension is different for the classic profile and high performance v-belts; however, they exhibit the same trend. This is confirmed by the shape parameter β in Table 5, which are similar.

The slip in power transmission systems that employ belts and pulleys, fits an inverse exponential model with 99% confidence; thus, it is reliable to use equations (14) and (15) to estimate the prestressed in belts from direct measurement of the slip, without requiring to turn off the machine.

According to the observations in Figure 6, the transmission systems with high performance v-belts are more efficient than the systems with v-belts of classic profile, because they exhibit a smaller slip.

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