



# RESISTANCE EFFECT ESTIMATION IN THE SHORT CIRCUIT CURRENT THROUGH A SENSITIVITY ANALYSIS

## ESTIMACIÓN DEL EFECTO DE LA RESISTENCIA EN LA CORRIENTE DE CORTOCIRCUITO MEDIANTE UN ANÁLISIS DE SENSIBILIDAD

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### Abstract

The assessment of the current in an electrical power system (EPS) after a fault, is generally termed short-circuit analysis. The magnitude of those currents is used for dimensioning the protection equipment of the EPS. Short-circuit analysis assumes that the electrical resistances of the components can be neglected, since they do not significantly affect the magnitude of the short-circuit currents. This work quantifies the effect of the electrical resistance of the elements of the EPS on the magnitude of the short-circuit current, by means of sensitivity (SA) and uncertainty (UA) analyses. The SA is based on the variance decomposition of an output variable, and can quantify the main effects (importance) and the interactions of the variables considered. On the other hand, The UA allows assessing how the variations in the variables considered affect the output. The proposed approach is illustrated on two networks from the literature, considering three-phase and single-phase faults. The results of such proposed approach numerically show that the effects due to taking into account the electrical resistance are indeed negligible, when compared to the rest of variables considered in the short-circuit analysis. This result coincide with the assumptions reported in the literature for the calculation of the fault currents.

**Keywords:** Short circuit current, electrical resistance, sensitivity analysis.

### Resumen

El cálculo de los valores de corriente que fluyen en un sistema eléctrico de potencia (SEP) posterior a una falla, se denomina análisis de falla o de cortocircuito. Los valores de la corriente de cortocircuito son empleados para el dimensionamiento de los equipos de protección del SEP. Los análisis de cortocircuito tienen como una de sus premisas despreciar la resistencia eléctrica de los elementos del sistema, pues esta no afecta en mayor medida las magnitudes de las corrientes de cortocircuito. En este trabajo se propone cuantificar el efecto de la resistencia eléctrica de los elementos del SEP en la magnitud de la corriente de cortocircuito, mediante un análisis de sensibilidad (AS) e incertidumbre (AI). El AS se basa en la descomposición de la varianza de una variable de salida y puede cuantificar los efectos principales (importancia) y las interacciones de las variables consideradas. Por otro lado, el AI permite evaluar cómo las variaciones en las variables consideradas afectan la salida. La propuesta se ilustra sobre dos redes de la literatura, considerando fallas trifásica y monofásica. El resultado de nuestra propuesta muestra numéricamente que los efectos debidos a considerar la resistencia eléctrica son de hecho insignificantes, en comparación con el resto de los factores que intervienen en el análisis de cortocircuito. El resultado coincide con las premisas de cálculo de las corrientes de falla supuestas en la literatura.

**Palabras clave:** corriente de cortocircuito, resistencia eléctrica, análisis de sensibilidad.

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## 1. Introduction

The short circuit studies are analyses that are used to determine the magnitude of the electrical currents that go through the electrical power systems (EPS), during a fault. Afterwards, such magnitudes are utilized to specify or validate the characteristics of the components of the system, such as breakers, buses, among others [1].

Short circuit studies in the EPS generally assume that the electrical resistance of the elements of the system is negligible, and exclusively consider their electrical reactance for calculating the magnitude of the short circuit current. According to [2], for most calculations of short circuit currents in medium and high voltage, and in some cases in low voltage, when the reactances are «much greater» than the resistances, is sufficiently accurate and simpler neglecting the resistances and considering only the reactances. Note that the rule is not specific, only suggests when the reactance is much greater than the resistance.

The same assumption for calculating the short circuit currents is made in the standard [3], where it is indicated that the calculation is much simpler, but with a loss of accuracy if the resistances are neglected, when the reactance/resistance (X/R) ratio is greater than 3.33.

The literature related to the analyses of EPS suggest similar procedures, regarding the possibility of neglecting the resistance of the elements. For example, it is indicated in [1] that it is possible to neglect the resistances in the fault study, because «it is not likely that they significantly influence in the level of the fault current».

A group of assumptions is presented in [4] for short circuit calculation, where it is suggested to neglect all the resistances of the elements (generators, transformers and transmission lines) to simplify the calculation.

Other works [5,6], establish 5 % as a maximum error in the value of the short circuit current, if the resistance of the elements of the system is neglected. This suggestion is derived from the general expression to determine the fault current, see equation (1):

$$|I_{cc}| = \frac{|V_{fault}|}{|R_{fault} + jX_{fault}|} = \frac{|V_{fault}|}{X_{fault} \times \sqrt{1 + \left(\frac{R_{fault}}{X_{fault}}\right)^2}} \quad (1)$$

Where:

- $I_{cc}$  = Short circuit current (A)
- $V_{fault}$  = Pre-fault voltage (V)
- $R_{fault}$  = Resistance at the fault point ( $\Omega$ )
- $X_{fault}$  = Reactance at the fault point ( $\Omega$ )

Where  $R_{fault}$  and  $X_{fault}$  correspond to the equivalent Thevenin impedance at the fault point. When

the ratio (X/R) is greater than 4, the error made by neglecting the resistance is smaller than 4 %. This is valid except for distribution or industrial systems where this ratio is smaller than 4 [6].

In power systems, the value of the resistance of the elements is usually very small compared to the value of their reactance. This consideration is the reason in which standards and authors are based for neglecting the resistances and their influence in the determination of the fault currents. In real power systems, the ratio X/R between the reactance and the resistance at the fault point is usually of an order between 15 and 120 times [7].

The references agree that the electrical resistance may be neglected in short circuit analyses, but there is no consensus regarding their effect in the magnitude of the short circuit currents.

In this work, the sensitivity analysis (SA) is used to quantify the effect of varying the resistance in the value of the short circuit current, for two networks in the literature. In addition, it is defined the relationship of these variations with the remaining parameters that enable determining the short circuit current (voltage and electrical reactance of the elements). The analysis is based on the use of the theory of SA and UA, and as a result of these analyses the same conclusion presented in the literature is directly reached, but from a different perspective.

The structure of the work is as follows: the first section presents some fundamental definitions associated to the sensitivity analysis; the second describes the procedure for determining the uncertainty associated to the short circuit current and the test electrical power systems; the third presents and discusses the results obtained and, finally, the fourth presents conclusions and future works.

### 1.1. Sensitivity and uncertainty analyses

According to [8], the uncertainty analysis (UA) is defined as the study of the amount of uncertainty contributed to the output of a model, by the different sources of uncertainty in the input. On the other hand, the sensitivity analysis (SA) evaluates the importance of the input variables of a model. Such importance is measured as a function of how much variability in the output of the model is due to the variability in the input variables. In this case, the uncertainty in the input variables is modeled through distribution functions with known parameters.

According to [8], the steps to carry out the SA/UA approach are defined as:

- Establish the objective of analysis, and accordingly define the form of the outputs of the model.
- Decide which input factors will be included in the analysis.

- Choose a probability distribution function for each of the input factors.
- Choose a method for SA, according to the characteristics of the problem under study.
- Generate the sample of the input factors. The sample is generated according to the specifications of the known parameters and the selected size of the sample.
- Evaluate the sample generated in the model and produce the corresponding outputs, which contain the values according to the form specified in step 1.
- Analyze the outputs of the model, determine the sensitivity indices (importance) and establish the conclusions.

The methods of SA can be classified according to the output of their measures: quantitative or qualitative, local (they do not allow varying all factors simultaneously) or global (they allow varying all factors simultaneously) and dependent or independent of the model [9].

Given a model  $Y = F(X_1, X_2, X_3, \dots, X_n)$  where  $Y$  is its output and  $X_i$  represent the input variables modeled as random variables (i.e., its uncertainty is modeled as a probability density function (pdf)), the variance  $V(Y)$  of the output  $Y$  may be described as in equation (2) [8]:

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \sum_i \sum_{j>i} \sum_{l>j} V_{ijl} + \dots + V_{12\dots k} \quad (2)$$

Where:

$V_i = V(E(Y|X_i))$  is the main effect (or of first order) due to  $x_i$

$V_{ij} = V(E(Y|X_i, X_j)) - V_i - V_j$  is the second order effect due to the interaction between  $x_i$  and  $x_j$ , and so forth.

The main sensitivity ( $S_i$ ) and total ( $S_{T_i}$ ) effects, may be defined as given in equations (3) and (4), respectively, according to [8]:

$$S_i = \frac{V_i(E_{-i}(Y|X_i))}{V(Y)} \quad (3)$$

$$S_{T_i} = \frac{E_{-i}(V_i(Y|X_{-i}))}{V(Y)} \quad (4)$$

Where:

$X_{-i} = (x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_k)$  and  $E_{-i}(Y | X_i)$

is the expected value of  $Y$  conditioned to  $x_i$ , and, therefore, is only a function of  $x_i$ .

The main index  $S_i$  is the fraction of the variance  $V(Y)$  of the output that can be attributed to  $x_i$  only, while  $S_{T_i}$  corresponds to the fraction of  $V(Y)$  that can be attributed to  $x_i$ , including all its interactions with the other input variables.

The main index  $S_i$  is the measure employed to determine the input variables that mainly affect the output uncertainty, while  $S_{T_i}$  is utilized to identify the subset of not influential input variables, i.e., those variables that can be fixed at any value in their uncertainty range, and they do not significantly affect the variance of the output [10].

The estimates of  $S_i$  y  $S_{T_i}$  are approximated: 1) assuming (statistical) independence between the input variables; 2) using particular sampling techniques to generate samples of the input variables; and 3) evaluating the group of samples obtained in 2) from the model under study [8].

There are different techniques for sensitivity analysis based on the decomposition of the variance; several of these techniques are mentioned in [10]. These techniques differ with respect to their computational complexity, as well as in the effects that they evaluate (main and/or total). Among these techniques, it should be mentioned: Sobol [11] that enables evaluating the main and total effects, and EFAST (Extended Fourier Amplitude Sensitivity Test) [12], an extension of FAST (Fourier Amplitude Sensitivity Test) [13], that also evaluates the main and total effects ( $S_i$  and  $S_{T_i}$ ), but with less computational complexity than the Sobol method.

## 2. Methodology

The following approach is proposed to estimate the effect of the electrical resistance of the elements (input variables of the model) on the short circuit current (output of the model):

A uniform distribution  $U[0 - 1, 2 \times \text{valor base}]$  is assumed for the input variables (pre-fault voltage, resistance and impedance of the elements of the power system). The distribution is asymmetrical and enables quantifying the effect of neglecting (values close to zero) the resistance of the elements of the power system.

After the evaluation of the described procedure (SA), a Monte Carlo simulation [14] is carried out considering only the variables of interest (electrical resistance of the elements). The Monte Carlo simulation is a method employed to evaluate the propagation of uncertainty through the generation of random variables. In this way, the propagation of the uncertainty is quantified at the output of the model, i.e., the vari-

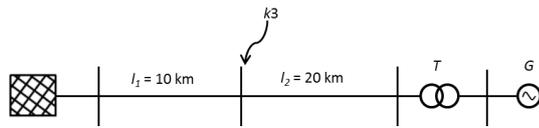
ation of the magnitude of the short circuit current is quantified.

The short circuit calculation and the SA/UA were carried out in the R free software [15]; specifically, the algorithms from the Sensitivity library were utilized for the SA.

## 2.1. Test electrical systems

### 2.1.1. Test power system 1 (TPS1)

The power system employed [16] is a nonmeshed network with two sources, as shown in Figure 1. It is constituted by an external system, two transmission lines, one transformer and one generator. It is assumed a solid three-phase to ground fault in  $k3$ .



**Figure 1.** Radial test power system (TPS1)

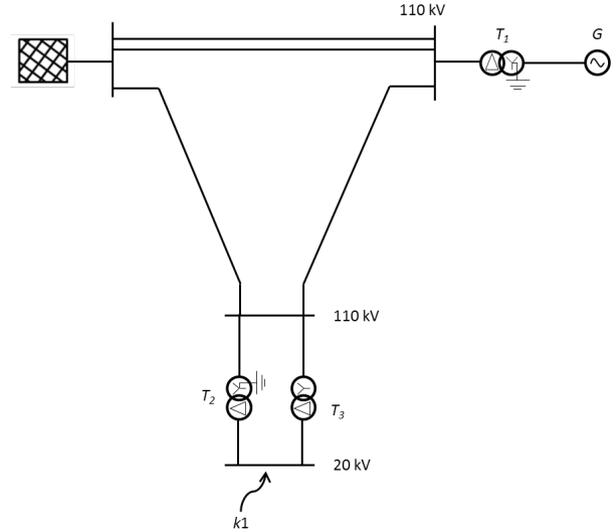
The elements of the radial test power system 1 (TPS1) are modeled; the values are shown in Table 1. These values represent the input variables of the model of SA.

**Table 1.** Impedance of the elements of the test power system 1 (TPS1)

N.º	Variable	Value
1	Pre-fault voltage	110 kV
2	Resistance external system	0,605 $\Omega$
3	Reactance external system	6,050 $\Omega$
4	Resistance line 1	1,930 $\Omega$
5	Reactance line 1	3,860 $\Omega$
6	Resistance generator	8,879 $\Omega$
7	Reactance generator	126,762 $\Omega$
8	Resistance transformer	2,710 $\Omega$
9	Reactance transformer	53,171 $\Omega$
10	Resistance line 2	2,440 $\Omega$
11	Reactance line 2	7,440 $\Omega$

### 2.1.2. Test power system 2 (TPS2)

The meshed electrical network taken from [16], for which it is assumed a solid one-phase to ground fault in  $k1$  (see Figure 2). The values of the elements are shown in Table 2, which are the input factors for the sensitivity analysis.



**Figure 2.** Meshed test power system (TPS2)

Since it is a one-phase fault, it is solved using the method of the sequence networks [1], and the elements should be modeled with their corresponding values of positive, negative and zero sequence. In order words, the parameters that take part in the SA are increased by 3. For example, for a line there will be a resistance of positive, negative and zero sequence.

**Table 2.** Impedance of the elements of the test power system 2 (TPS2)

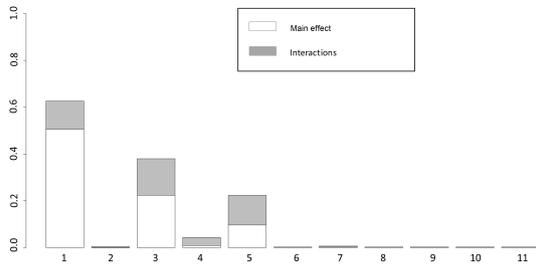
N.º	Variable	Value	Nº	Variable	Value
1	Pre-fault voltage	20 kV	24	Resist. Line 3_0	0,161 Ω
2	Resist. External System_0	0,030 Ω	25	React. Line 3_0	0,648 Ω
3	React. External System_0	0,300 Ω	26	Resist. Line 3_+	0,061 Ω
4	Resist. External System_+	0,020 Ω	27	React. Line 3_+	0,199 Ω
5	React. External System_+	0,200 Ω	28	Resist. Line 3_-	0,061 Ω
6	Resist. External System_-	0,020 Ω	29	React. Line 3_-	0,199 Ω
7	React. External System_-	0,200 Ω	30	Resist. Transf. 1_0	0,147 Ω
8	Resist. Generator_+	0,224 Ω	31	React. Transf. 1_0	2,746 Ω
9	React. Generator_+	3,200 Ω	32	Resist. Transf. 1_+	0,147 Ω
10	Resist. Generator_-	0,224 Ω	33	React. Transf. 1_+	2,746 Ω
11	React. Generator_-	4,800 Ω	34	Resist. Transf. 1_-	0,147 Ω
12	Resist. Line 1_0	0,084 Ω	35	React. Transf. 1_-	2,746 Ω
13	React. Line 1_0	0,374 Ω	36	Resist. Transf. 2_0	0,027 Ω
14	Resist. Line 1_+	0,016 Ω	37	React. Transf. 2_0	0,761 Ω
15	React. Line 1_+	0,064 Ω	38	Resist. Transf. 2_+	0,027 Ω
16	Resist. Line 1_-	0,016 Ω	39	React. Transf. 2_+	0,761 Ω
17	React. Line 1_-	0,064 Ω	40	Resist. Transf. 2_-	0,027 Ω
18	Resist. Line 2_0	0,161 Ω	41	React. Transf. 2_-	0,761 Ω
19	React. Line 2_0	0,648 Ω	42	Resist. Transf. 3_+	0,027 Ω
20	Resist. Line 2_+	0,061 Ω	43	React. Transf. 3_+	0,761 Ω
21	React. Line 2_+	0,199 Ω	44	Resist. Transf. 3_-	0,027 Ω
22	Resist. Line 2_-	0,061 Ω	45	React. Transf. 3_-	0,761 Ω
23	React. Line 2_-	0,199 Ω			

Note: The symbols +, -, 0 indicate the values of positive, negative and zero sequence, respectively.

### 3. Results

#### 3.1. Test power system 1 (TPS1)

Figure 3 shows the sensitivity indices of first order ( $S_i$ ) (white part of the bars) and total ( $S_{T_i}$ ) (complete bar, white and gray parts) for the TPS1. The variables that most affect the short circuit current are, in order of importance: the pre-fault voltage ( $S_1 = 0.456$ ), the reactance of the external system ( $S_3 = 0.252$ ) and the reactance of line 1 ( $S_5 = 0.108$ ). The remaining factors, including the resistance of the elements, have very small values of importance  $S_{T_i}$ , and consequently their effects may be considered negligible.

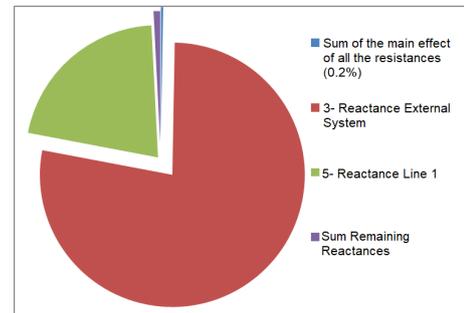


**Figure 3.** Main ( $S_i$ ) and total ( $S_{T_i}$ ) effects of the input variables in the short circuit current of the TPS1

As it was mentioned, the pre-fault voltage has a very high importance (0.456); due to this, it is subsequently fixed as a constant in the model, in order to

only evaluate the effects of the reactances and resistances of the elements; likewise very small effects were obtained for the resistances.

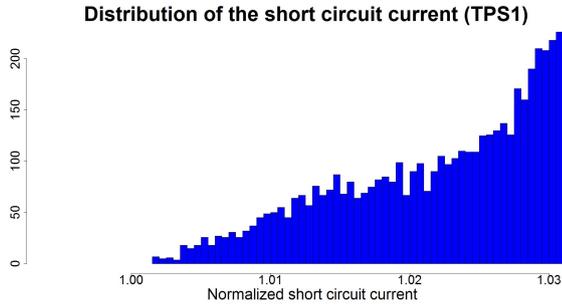
For comparison purposes, all the main effects of the resistances and the main effects of the less important reactances were grouped (see Figure 4), keeping constant the pre-fault voltage. The sum of the main effect of the resistances is negligible compared to the main effect of the reactances, in this system and, for this particular fault, the most important variable is the reactance of the external system.



**Figure 4.** Main effects of the reactances and sum of the main effects of the resistances.

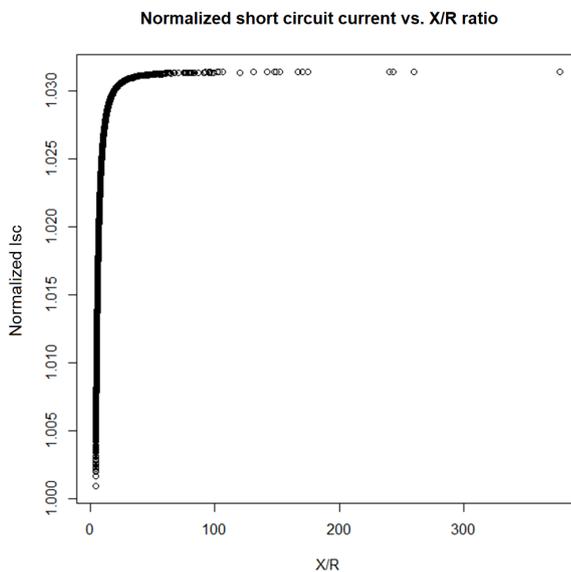
After evaluating the SA, and only considering the resistances of the elements, the evaluation of the Monte Carlo method [14] is carried out, to obtain the approx-

imate probability distribution of the short circuit current for 5000 evaluations. The short circuit current for the values of Table 1 is 7196 A, which corresponds to the normalized value of 1 unit, in the approximate histogram of the short circuit current presented in Figure 5. The minimum and maximum values obtained for the short circuit current were 0.995 and 1.031, respectively. The average value of this distribution is 1.022, and the shape of the distribution is asymmetrical, with a bias to the upper end.



**Figure 5.** Approximate histogram of the short circuit current (TPS1)

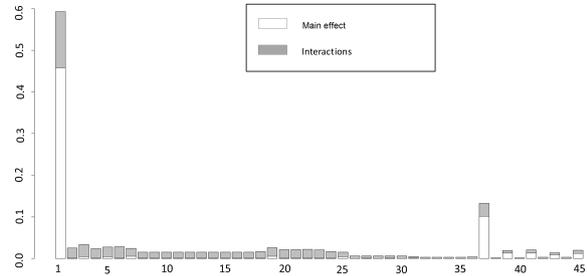
The literature refers to the X/R ratio to neglect the electrical resistance in the short circuit calculations. For this reason, Figure 6 shows the short circuit current normalized for the values of X/R obtained in the Monte Carlo simulation. The values of the X/R ratio vary from approximately 4 to 400; considering this wide range, the normalized short circuit current does not exhibit variations achieving at least 4 %.



**Figure 6.** Short circuit current normalized for the values of X/R (TPS1).

### 3.2. Test power system 2 (TPS2)

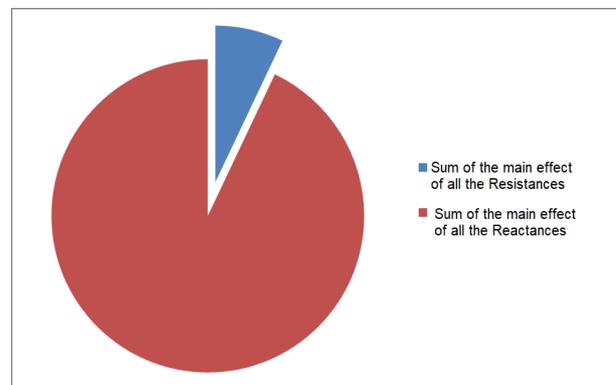
The results obtained for the meshed power system TPS2 are similar to those obtained for the TPS1. The pre-fault voltage turns out to be the most important variable (see Figure 7). In this system, there is more uncertainty of superior order associated to the interaction of variables ( $S_{T_i} - S_i$ ), which is due to the fact that the system is meshed, and various equivalent impedances (successive sums and products) should be calculated to obtain the short circuit equivalent impedance.



**Figure 7.** Main ( $S_i$ ) and total ( $S_{T_i}$ ) effects of the input variables in the short circuit current of the TPS2.

The numbers of the variables (x-axis) of Figure 7, are in accordance with the numbering of Table 2. Note that the variable 37 (zero sequence reactance of transformer 2) appears as the second most important variable, even though with a very small contribution.

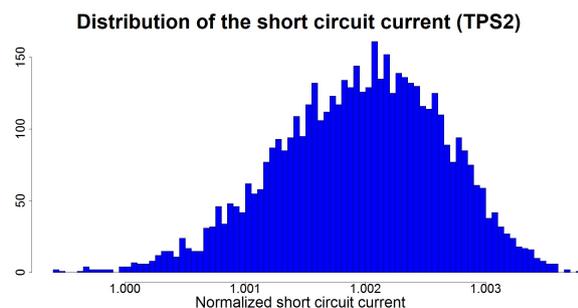
Figure 8 shows a comparison of the sum of the total indices of the resistances and of the reactances of the system TPS2, considering constant the pre-fault voltage; the percentage represented by the index of the sum of the resistances is slightly smaller than 7 %.



**Figure 8.** Sum of the main effects of the resistances and reactances of the TPS2

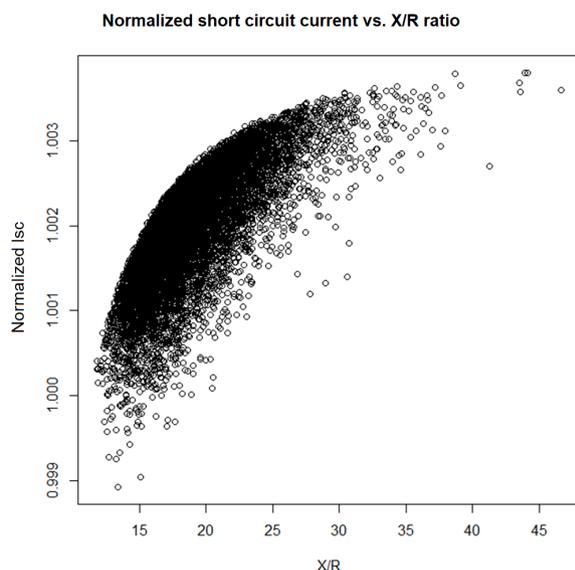
Figure 9 shows the approximate histogram for the short circuit current, obtained using the Monte Carlo technique disturbing only the resistances of the elements of the system TPS2. For the values in Table 2 this current is 13679 A, which corresponds to the

normalized value of 1 unit, in the histogram of Figure 9. The minimum and maximum values obtained for the short circuit current were 0.9999 and 1.0037, respectively. Note that carrying out variations in the resistances, the effect on the short circuit current is negligible. This distribution is more symmetrical than the one in the previous example, and has an average value of 1.002.



**Figure 9.** Approximate histogram of the short circuit current (TPS2)

Figure 10 shows the short circuit current normalized for the values of  $X/R$  obtained in the Monte Carlo simulation, for the TPS2 system. In this case the variation is much smaller, since the normalized short circuit current does not exhibit variations achieving 1 %.



**Figure 10.** Short circuit current normalized for the values of  $X/R$  (TPS2)

## 4. Conclusions

In this work, it is estimated the uncertainty of the short circuit current due to the electrical resistance of the elements, through an approach of sensitivity and

uncertainty. This analysis can be applied to any other power system with any location and/or type of fault.

For the two cases under consideration, the results are in accordance with what is suggested by different authors: it is possible to neglect the resistance of the elements, whenever their reactance is much greater than their resistance ( $X/R$  ratio). The variation of the magnitude of the short circuit current in the two cases evaluated does not exceed 4 %, which coincides with the value of 4-5 % reported in some consulted works [5, 6].

This analysis not only considers the uncertainty in the short circuit current, due to the uncertainty in the resistances and reactances, but also enables quantifying the total variation, i.e., the percentage due to the uncertainty of the resistances of the components of the EPS. In these two cases, the effect of the resistance of the elements is approximately 7 % (in the test power system 2 (TPS2)); the rest is associated to the reactances, if the value of the pre-fault voltage is considered constant, for the sensitivity analysis carried out.

The results obtained in these two systems, could be extrapolated to real power systems in medium or high voltage, because in such systems the reactance is usually much greater than the resistance ( $X \gg R$ ). As a future work, it is intended to establish ratios and/or critical values between the reactance/resistance factor ( $X/R$ ) at the fault point of the system, and the main indices ( $S_i$ ) of the uncertainty of the short circuit current.

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