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Evaluation of the dynamic properties of a 2D-frame (MDOF) in a shake table

Evaluación de las propiedades dinámicas de un pórtico plano (MDOF) en una mesa vibratoria

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Abstract

At present, the study of structural dynamics is mainly theoretical with access to certain simulations through software, however, this project attempts that the student may understand and physically observe the dynamic responses of experimental models. These models correspond to 2D - frames with multiple degrees of freedom that are subjected to acceleration in the base; this acceleration is generated by a Shake Table II, and the results obtained will be compared with theoretical results. These theoretical results were obtained based on modal decomposition and Newmark's method for calculating the dynamic response, considering the linear variation in the acceleration of each floor. The application developed, ATH Dynamic Responses, provided the theoretical responses through a graphical interface friendly for the user. The experimental models are constituted by two materials: stainless steel for frame legs and acrylic sheets for floors; these were tested on a Shake Table II". The data was acquired using accelerometers that were placed in each floor and in the shake table, and they were corrected both by baseline and with the low pass filter. The results obtained show that the instrumentation with the Shake Table II and the data acquisition with accelerometers provide results similar to the theoretical ones regarding dynamic responses and modal properties.

Keywords: modal decomposition, structural dynamics, experimental model, Shake Table, Newmark, filter

Resumen

Hoy en día, el estudio de la dinámica estructural es fundamentalmente teórico con acceso a ciertas simulaciones vía *software*, sin embargo, este proyecto intenta que el estudiante pueda entender y observar de manera física las respuestas dinámicas de modelos experimentales. Estos modelos corresponden a pórticos planos de múltiples grados de libertad, que están sometidos a aceleración en la base, la cual es generada por una mesa vibratoria. Los resultados obtenidos se compararán con teóricos. Estos fueron obtenidos basándose en la descomposición modal y en el método de Newmark para el cálculo de la respuesta dinámica, considerando variación lineal en la aceleración de cada piso. La aplicación generada, ATH Dynamic Responses, proporcionó las respuestas teóricas, mediante una interfaz gráfica amigable para el usuario. Los modelos experimentales están constituidos por dos materiales: acero inoxidable (parantes) y láminas de acrílico (pisos), y fueron ensayados sobre una mesa vibratoria. La adquisición de datos se realizó mediante acelerómetros que se colocaron en cada piso y sobre la mesa vibratoria, fueron corregidos, tanto por línea base como con el filtro pasa bajo. Los resultados obtenidos muestran que la instrumentación con una mesa vibratoria y adquisición de datos con acelerómetros proporcionan valores muy similares a los teóricos en cuanto a respuestas dinámicas y propiedades modales.

Palabras clave: descomposición modal, dinámica estructural, modelo experimental, mesa vibratoria, Newmark, filtro

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1. Introduction

Ecuador has a significant seismic activity, the last earthquake that affected the country with great intensity occurred on April 16th, 2016 with a magnitude of momentum M_w of 7.8, it happened in the coastal provinces of Manabí and Esmeraldas [1], causing material damages, collapsing of buildings and nearly seven hundred people dead.

The earthquake was caused by the subduction of the Nazca oceanic plate below the South American plate; the friction between both plates produces an accumulation of elastic energy, which is relaxed when there is a sudden rupture and the seismic event occurs [2].

This has motivated engineers to develop a philosophy which is centered in preventing life losses, controlling the collapse of all structures [3]. As a result, every building should be designed considering seismic solicitation; the analysis of the structure when facing this type of load is the main problem that structural dynamics seeks to solve.

In most cases, the study of structural dynamics is carried out theoretically, without being able to physically observe the behavior of structures in the event of an earthquake or acceleration in the base. Therefore, this project is focused on constructing experimental models with multiple degrees of freedom and made of appropriate materials, which means slabs very rigid compared with the bending columns, where the model will concentrate the deformation of the structure. In addition, both columns and floors are considered axially rigid, in the analytical model, it will be considered that floors concentrate the mass, and that frame legs are the ones that collaborate with the rigidity, i.e., that it behaves as a shear building [4].

Experimental techniques, such as placing triaxial accelerometers or the use of dynamic excitation equipment that reproduce earthquakes to scale [5], have been implemented to obtain dynamical responses and modal properties in experimental models and real structures. A shake table, known as Shake Table II [6], was used in the project. This device is an earthquake simulator for small physical-academic models (maximum mass = 7.5 kg), that will enable generating floor accelerations in the form of pulsations, sinusoidal sweeps and seismic records (scaled) [6]. The Shake Table II enables reproducing floor accelerations in two directions (x,y), however, for the project it was used 2-D models and unidirectional acceleration, each floor will move in only one direction [7].

Dynamic properties of experimental structures have been determined in [7,8] and [9] using the same method, however, these are centered only in maximum displacements and others only in obtaining frequencies; besides, they are limited to two floors. In the project it will be analyzed how frequencies and dampings are obtained and, besides, models according to their construction mode, have the possibility of stacking until becoming a model of six floors, and the unique limitation is the maximum weight withstood by the shake table.

2. Materials and methods

The project consists of the theoretical foundation, which includes the concepts about structural dynamics used to develop the Matlab script; and the experimental part, where it is established the type of material to be used, the geometrical features, the acceleration of the base, the acceleration of each floor, the operation of the shake table, and the way to acquire and process the responses.

2.1. System with multiple degrees of freedom

From the point of view of structural dynamics, a system with multiple degrees of freedom (MDOF) is one that requires more than one coordinate to describe its movement. The degrees of freedom may also determine the directions of acceleration of the concentrated masses. In the project, the direction of the earthquake will be uniaxial, and hence, the number of degrees of freedom will be one per floor [10].

Experimental models are modular and may become systems from one to six degrees of freedom, the tests were carried out with systems of one, two, and three degrees of freedom. Figure 1 shows the model with three degrees of freedom.

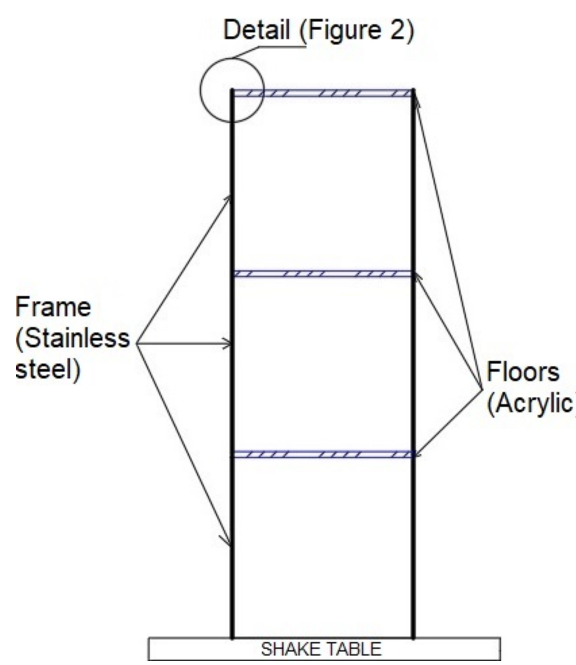


Figure 1. Model with three degrees of freedom

2.2. Shear building

A shear building is characterized as an array that concentrates the mass in each floor, and besides they should act as diaphragms infinitely rigid to bending and axial load. Therefore, in the model, only the columns should collaborate with the rigidity [4].

The preceding considerations enable simplifying the structure and solving the problem as an MDOF, where the slabs are infinitely rigid, and enable assuring that there will be no rotations between frame legs and floors. In the experimental model, the rotation was controlled according to the connection between the frame leg and the acrylic, because the connection is not fixed at a point but in an area, as observed in Figure 2. Regarding axial deformation, it will be negligible due to the physical features of the floors.

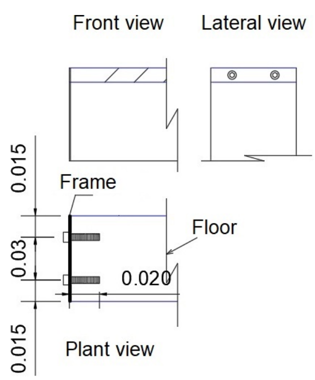


Figure 2. Detail of the connection

In the experimental model it should be taken into account that columns add mass, and therefore it was considered that the slab concentrates half of the mass of each column that is above and below it, as shown in Figure 3.

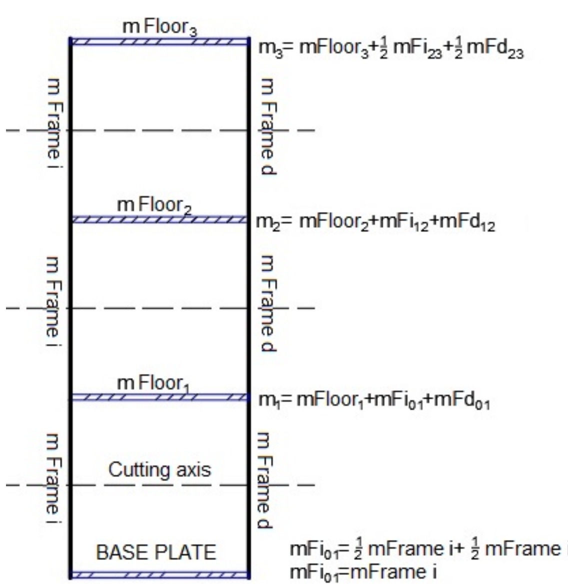


Figure 3. Distribution of mass of the structure

Where: mFi_{ij} and mFd_{ij} is the mass of the frame leg from degree of freedom i to degree of freedom j.

Cutting axis: helps to quantify how the mass of each floor was concentrated.

2.3. Inertia force

It relates the external forces that act on the mass of the structure with the accelerations of the dynamic degrees of freedom, as shown in Equation (1) [11].

$$\{Fi\} = [M] \times \{\ddot{x}\} \tag{1}$$

Where $\{Fi\}$ is the vector of inertial force, $[M]$ is the mass matrix and $\{\ddot{x}\}$ is the vector of acceleration of the degrees of freedom.

The mass matrix (2) is an estimation of the mass of the whole structure, this matrix is also known as «matrix of concentrated mass» [12].

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \tag{2}$$

2.4. Elastic force

It relates the external forces on the rigidity of the structure with the displacements of the dynamic degrees of freedom, as shown in Equation 3 [11].

$$\{Fs\} = [K] \times \{x\} \tag{3}$$

Where $\{Fs\}$ is the vector of elastic force, $[K]$ is the rigidity matrix and $\{x\}$ is the vector of displacement of the degrees of freedom.

The rigidity matrix ((4) includes properties of the columns, such as length, area and modulus of elasticity [12].

$$[K] = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1N} \\ k_{21} & k_{22} & \cdots & k_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ k_{N1} & k_{N2} & \cdots & k_{NN} \end{bmatrix} \tag{4}$$

2.5. Damping force

It is a mechanism with which energy may be dissipated from the structure; according to this, the external forces that act on the damping are related with the velocities of the dynamic degrees of freedom as shown in Equation **??**5 [11].

$$\{Fd\} = [C] \times \{\dot{x}\} \tag{5}$$

Where $\{Fd\}$ is the vector of damping force, $[C]$ is the damping matrix and $\{\dot{x}\}$ is the vector of velocity of the degrees of freedom.

Classical damping will be supposed for obtaining the damping matrix. The classical damping matrix may be used in this type of models, if the damping mechanisms are similar in all the structure, i.e., a MDOF structure which in turn is constituted by the same structural system and similar materials, along the entire height [11].

One of the procedures within classical damping is modal damping. The analysis provides the damping of a specific number of modes, as indicated in expression (6) [13] y [14].

$$\{\phi_n\}^T \times [C] \times \{\phi_n\} = 2 \times \xi_n \times \omega_n \tag{6}$$

Where C is the damping matrix, ϕ_n is the vector of each modal form, ξ_n is the damping factor, ω_n the natural frequency, and n is the number of modes.

2.5.1. Damping factor

An important feature is the damping of the structure, which is defined based on the damping factor; for this reason, it should be obtained applying the bandwidth method described in the following [15].

a) Bandwidth

It is a method to obtain the damping factor in the frequency domain; this technique is widely used in professional practice, in which a structure should be excited by simultaneous or individual pulsations at different frequencies [16].

With the purpose of applying the method it should be considered the effect of the movement of the base, whereby it is proceeded to find the ratio between the amplitudes of the Fourier transform of the acceleration records of each slab with respect to the records of the base. This is known as transmissibility [11], which is indicated in expression (7).

$$Tr = \frac{\alpha_o(\omega)}{\alpha_b(\omega)} \tag{7}$$

Where Tr is transmissibility, $\alpha_o(\omega)$ amplitude of the acceleration of each floor in the frequency domain and $\alpha_b(\omega)$ is amplitude of the acceleration of the base in the frequency domain.

From the calculation of the transmissibility, plots with respect to the frequency are obtained and the damping factor is determined by means of the difference between two frequencies called medium power points at the frequency corresponding to each mode. The medium power points are the frequencies located at $\frac{1}{\sqrt{2}}$ of the maximum amplitude of the transmissibility, as indicated in Figure 4, and this will enable applying Equation (8) [17].

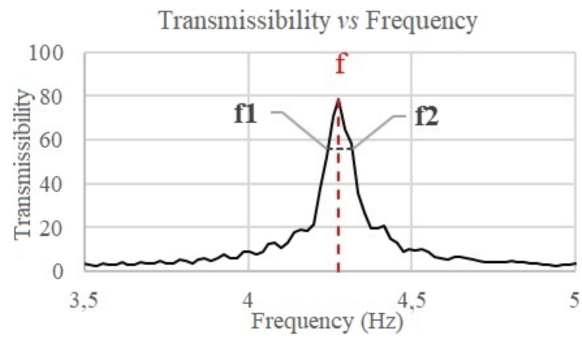


Figure 4. Bandwidth

$$\xi = \frac{f_2 - f_1}{2 \times f} \tag{8}$$

Where f is the frequency with maximum amplitude (Hz), f_1 and f_2 are the medium power frequencies (Hz).

2.6. Equation of motion

Based on the forces detailed in the previous sections, the equation of the MDOF system will be established, taking into account Newton´s second law of motion, it is obtained Equation (9).

$$[M]\times\{\ddot{u}\}+[C]\times\{\dot{u}\}+[K]\times\{u\} = -[M]\times\{\iota\}\times\{\ddot{x}o\} \tag{9}$$

Where $\{\iota\}$ is the positioning vector, $\{\ddot{x}o\}$ the floor acceleration for each time instant and u is the relative coordinate of each degree of freedom with respect to the base.

Figure 5 presents the forces acting on the system.

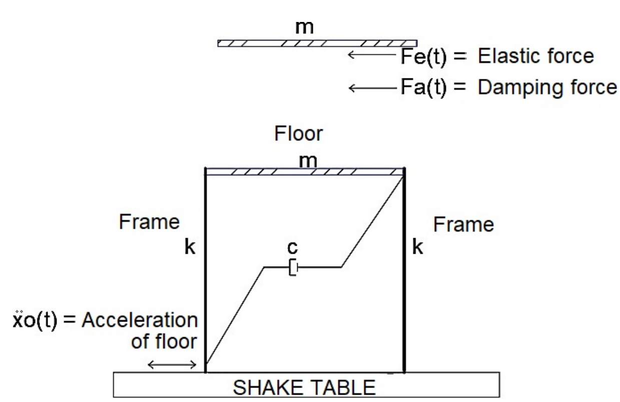


Figure 5. Acting forces

2.7. Newmark´s method

It is a method very versatile for calculating the response of a dynamic system. In 1959, N. M. Newmark developed a family of methods depending on the law of variation between consecutive time instants [11]; the linear acceleration method was the one used here, as shown in Figure 6.

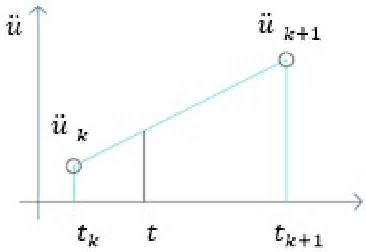


Figure 6. Linear acceleration

The matrix representation of the method applied for obtaining the responses of acceleration, velocity and position in time for a system with one degree of freedom, is shown in expression (10), where the response at instant $k+1$ only depends on the response at instant k [11].

$$\begin{pmatrix} u_{k+1} \\ \dot{u}_{k+1} \\ \ddot{u}_{k+1} \end{pmatrix} = [A] \times \begin{pmatrix} u_k \\ \dot{u}_k \\ \ddot{u}_k \end{pmatrix} + [B] \times \ddot{x}o_{k+1} \tag{10}$$

Where $[A]$, $[B]$ are constant matrices that depend on the frequency, damping, rigidity and time interval, and $\ddot{x}o_{k+1}$ is floor acceleration at instant $k+1$ [12] and [18]

2.7.1. Stability of the method

There are two types of methods depending on their stability: conditionally stable and unconditionally stable [11].

Conditionally stable procedures depend on the passage of time and unconditionally stable procedures are independent of the passage of time [11].

Newmark´s method based on linear acceleration is conditionally stable, and must comply with expression (11) to make possible its application.

$$\frac{\Delta t}{T} < 0,551 \tag{11}$$

Where Δt is the time interval and T is the period of the system.

2.8. Modal analysis

It enables calculating the response of the structure based on the vibration modes. For this purpose, it is important to define its dynamic properties: frequency, damping and modal shapes for each mode [19].

This analysis is very useful because it enables decomposing the responses of a structure in models of one degree of freedom, and combine them to obtain the response of the MDOF system.

2.8.1. Vibration frequencies and modes

In the damped system with multiple degrees of freedom, it should be established natural frequencies and modal shapes, considering it as a system subject to free vibration and without damping, such that in expression (9) the terms floor acceleration and damping become zero, thus resulting in expression (12).

$$[M] \times \ddot{u} + [K] \times \{u\} = 0 \tag{12}$$

Solving this differential equation yields expression (13), which is the base for solving the eigenvalues and eigenvectors problem [11].

$$\left[[K] - [M] \times \left\{\omega^2\right\}\right] \times \{\phi\} = 0 \tag{13}$$

Equation (13) corresponds to a system of simultaneous homogeneous equations, which by definition only has a non-trivial solution, therefore, the determinant of the coefficient matrix is zero, as indicated in Equation (14) [20].

$$\left|[K] - [M] \times \left\{\omega^2\right\}\right| = 0 \tag{14}$$

When expanding the determinant, it is obtained a polynomial of order 2n (n: number of vibration modes), where ω^2 is the variable. This equation is known as «equation of frequencies». The solutions are called eigenvalues, and their square roots correspond to the natural frequencies ω of the system [21].

In order to determine the eigenvectors or vibration modes ϕ, the natural frequencies ω are substituted in equation (13), and such equation is solved.

2.8.2. Orthogonality of the modes

The previous analysis enables demonstrating that the vibration modes corresponding to different frequencies fulfill the orthogonality condition, which is shown in the following equation (15) [11].

$$\begin{gathered} \{\phi i\}^T \times [M] \times \{\phi j\} = 0 \\ \{\phi i\}^T \times [K] \times \{\phi j\} = 0 \\ i \neq j \; Where \; i, j: \; vibration \; modes \end{gathered} \tag{15}$$

Where ϕi y ϕj are modal shapes for the modes i and j, $[M]$ the mass matrix and $[K]$ the rigidity matrix.

From this it is obtained that the system may be solved for each vibration mode as a separate system, without the influence of one mode with respect to the other, and the response of the entire system is defined based on expression (16).

$$\begin{aligned} \{x\} &= \{\phi 1\} \times q1(t) + \{\phi 2\} \times q2(t) + \cdots \{\phi n\} \times qn(t) \\ \{\dot{x}\} &= \{\phi 1\} \times \dot{q1}(t) + \{\phi 2\} \times \dot{q2}(t) + \cdots \{\phi n\} \times \dot{qn}(t) \\ \{\ddot{x}\} &= \{\phi 1\} \times \ddot{q1}(t) + \{\phi 2\} \times \ddot{q2}(t) + \cdots \{\phi n\} \times \ddot{qn}(t) \end{aligned} \tag{16}$$

Where ϕ_i is modal shape, and q, $\dot{q}$ and $\ddot{q}$ are the modal coordinates of position, velocity and acceleration, respectively, for the i-th mode.

Replacing the response $\{x\}$, $\{\dot{x}\}$ and $\ddot{x}$ based on the sum of the modes, and pre-multiplying by the transpose of the modal shapes matrix, it is obtained equation (17).

$$[\Phi]^T [M] [\Phi] \times \ddot{q}_n + [\Phi]^T [C] \times [\Phi] \times \dot{q}_n + [\Phi]^T \times [K] \times [\Phi] \times q_n = - [\Phi]^T \times [M] \times \{\iota\} \times \ddot{x}o \tag{17}$$

Based on equation ([**?**]) and the property of orthogonality, it may be defined equation (18) for each vibration mode.

$$[\phi_n]^T [M] [\phi_n] \times \ddot{q}_n + [\phi_n]^T [C] \times [\phi_n] \times \dot{q}_n + [\phi_n]^T \times [K] \times [\phi_n] \times q = - [\phi_n]^T \times [M] \times \{\iota\} \times \ddot{x}o \tag{18}$$

The relative dynamic responses (position, velocity and acceleration) are obtained at each time instant for each modal coordinate, using equation (18) and with the help of Newmark´s method of linear acceleration.

2.9. Design of the program

2.9.1. Matlab programming language

The ATH Dynamic responses program has been developed in the software Matlab, which is a programming language with a friendly working environment. It enables working in console mode (it only presents results based on expressions introduced) and in routine mode (programs with coded commands and enables to make and store programs) [10] and [22].

The most important Matlab feature is that it enables handling vectors and matrices directly, and besides the coding is not complex [10].

The project was made in routine mode, since a program with coded commands was run and a GUIDE created, which enabled improving the graphical user interface.

2.9.2. ATH Dynamic responses program

The ATH Dynamic responses program is based on Newmark´s numerical method considering linear acceleration for calculating the response of systems with 1 DOF in modal analysis.

Geometrical features, rigidity of the frame legs and mass considered as concentrated in each floor should be entered into the program. Based on this it is obtained relative dynamic responses (position, velocity and acceleration) and modal properties of the physical model (frequencies, modes and percentage of participative mass).

In order to understand the functionality of the program, Figure 7 indicates its flow diagram.

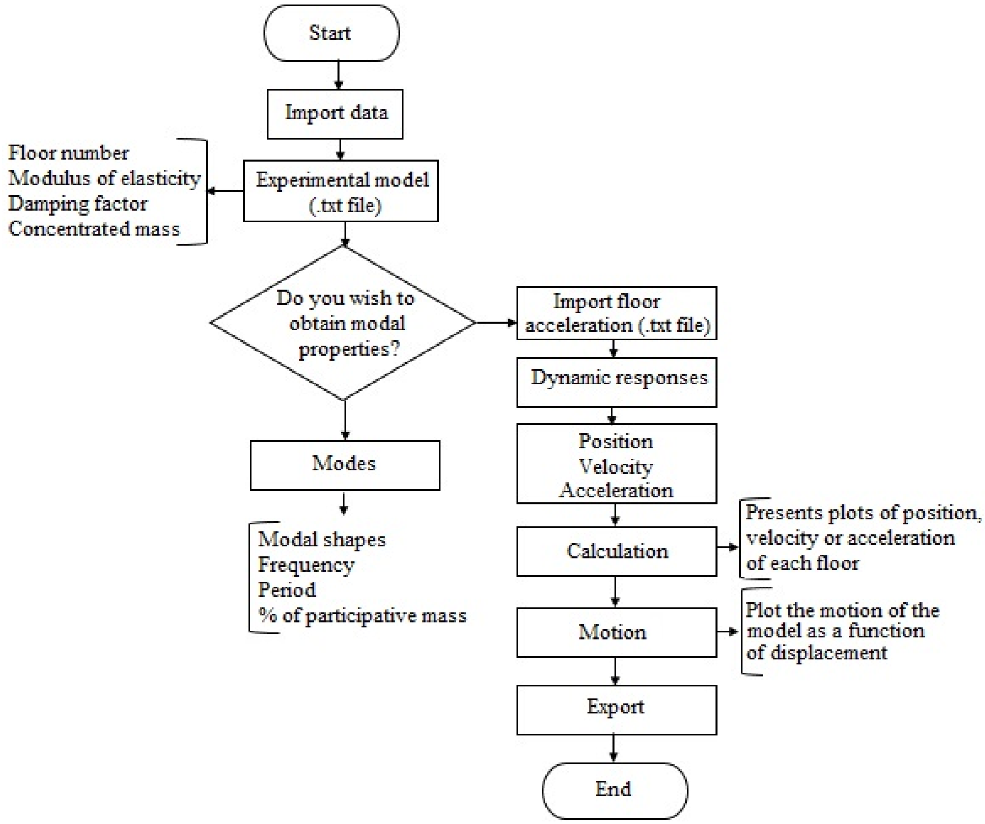


Figure 7. Flow diagram of the ATH Dynamic responses program

2.10. Data processing

The data obtained by the accelerometers need to be corrected by baseline and by filtering unwanted frequencies. To achieve this, it was used a low pass Butterworth filter [23], with a cut-off frequency of 16 Hz. The AB signal program [24] was used for this purpose.

Table 1. Properties of the accelerometers

Type	Frequency	Sensitivity
Normal accelerometer	2 Hz – 5 kHz	100 m V/g
Normal accelerometer	1 Hz – 5 kHz	100 m V/g
Miniature accelerometer	1 Hz – 4 kHz	100 m V/g

2.11. Instrumentation

In general, the instrumentation to measure the dynamic response of a structure consists of the installation of sensors that record dynamic responses (velocity, acceleration and displacement) [25].

The purpose of the instrumentation is registering the response in front of displacement, internal motions, earthquakes and observing the behavior of structures or models [24].

The acquisition of the project data was carried out by means of PCB Piezotronics accelerometers [25], which have the frequencies and sensitivities shown in Table 1.

2.12. Shake Table II

The test in a shake table is the most direct way to simulate the dynamic behavior of structures. The models will be limited to be not very heavy, of scaled dimensions and not very rigid [26].

Shake Table II was originally developed by the University Consortium on Instructional Shake Tables (UCIST) [6]. It is a mechanical device, which consists of an upper plate of 45.7 × 45.7 cm^2 where the model is anchored, a lower plate of 60,9 × 45,7 cm^2 and a DC motor with a power of 400 W. The table withstands a load of 7.5 kg and an acceleration of 2.5 g and enables motions with displacements up to ±7.62 cm [6].

2.13. Materials

The equipment used for carrying out this project is constituted by a Shake Table II [6] and accelerometers PCB Piezotronics [25], described in sections 2.11 and 2.12.

The experimental models are constituted by acrylic and stainless steel. The features and dimensions of the materials are specified in Tables 2 and 3.

Table 2. Specifications of the materials for frame legs and floors

Material	Thickness (m)	Width (m)	Lengh (m)	I (m^4)	E (kN/m^2)
Frame Stainless Steel	$7E^{-4}$	0,06	0,3	$1{,}715E^{-12}$	$1{,}80E^{+11}$
Floor acrylic	0,01	0,06	0,31		

Table 3. Mass of frame legs and floors

	Material	Mass (g)
Floors	Acrílico	232-234
Frame model 1	Acero inoxidable	102
Frame model 2	Acero inoxidable	204
Frame model 3	Acero inoxidable	299

Figure 8 presents drawings of the experimental models that were built. Figure 9 indicates the dimensions of the model with 3 degrees of freedom and, besides, how the mass concentration for the theoretical model was made, based on the experimental model, and Figure 10 shows an image of the real model.

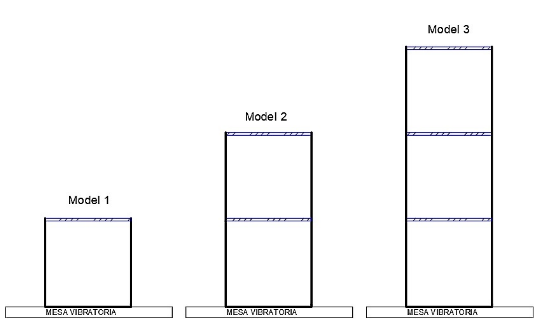


Figure 8. Models of one, two and three floors

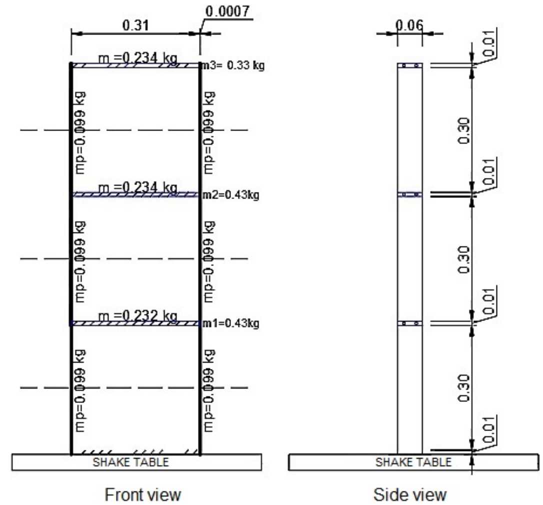


Figure 9. Dimensions of the model with 3 degrees of freedom and mass concentration

Figure 10. Experimental model of the model with 3 degrees of freedom

3. Results and discussion

Results show frequencies, damping factors, absolute accelerations of the experimental models and their comparison with the theoretical results provided by the ATH Dynamic responses program.

3.1. Experiment 1. Determining the frequencies and damping factors of each model

The frequencies were obtained theoretically using the ATH Dynamic responses program and verified with the SAP2000 software [27]. For obtaining the experimental frequency and the damping factor, a sweep of frequencies was performed with the Shake Table II.

It was carried out a baseline and filtering correction process, as explained in section 2.10. Based on this, the fast Fourier transform available in Matlab [23] was used, and transmissibility vs. frequency plots were obtained, as shown in Figures 11, 12 and 13, where the abscissa of each peak in the plot corresponds to the frequency of each mode and, besides, it enables obtaining the damping factor applying equation (8). The percentage error in the analytical frequency was obtained with respect to the experimental acceleration.

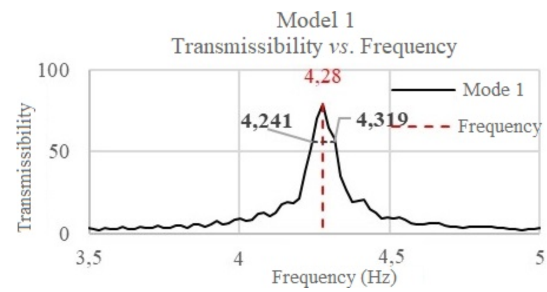
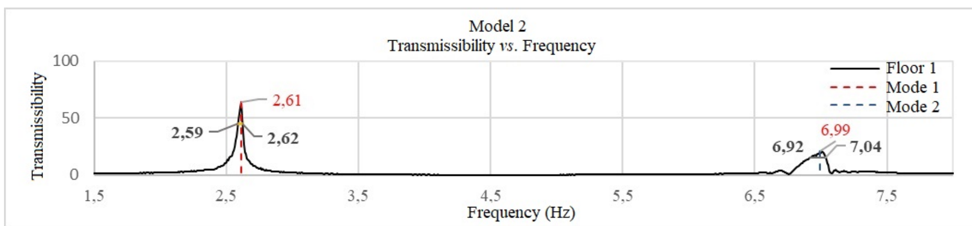
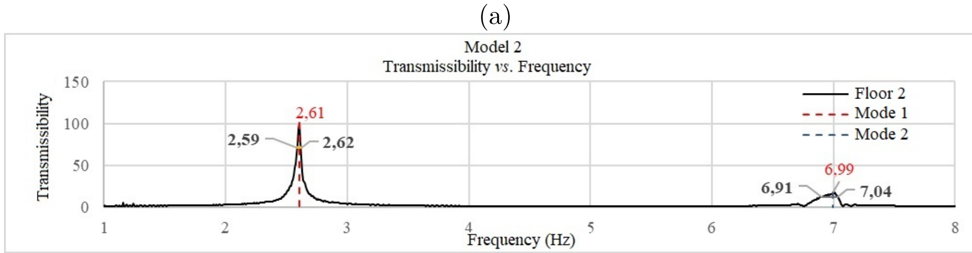


Figure 11. Frequency and bandwidth for the model

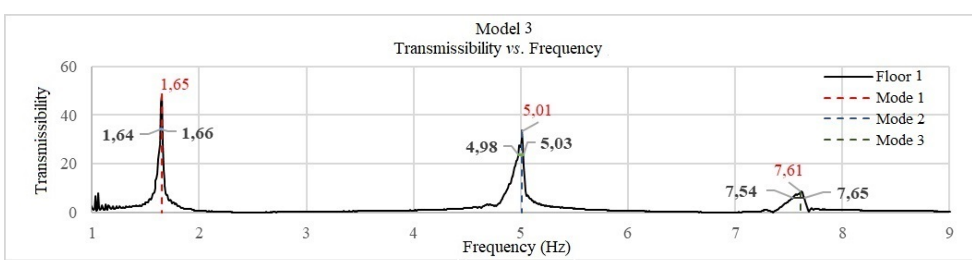


(a)

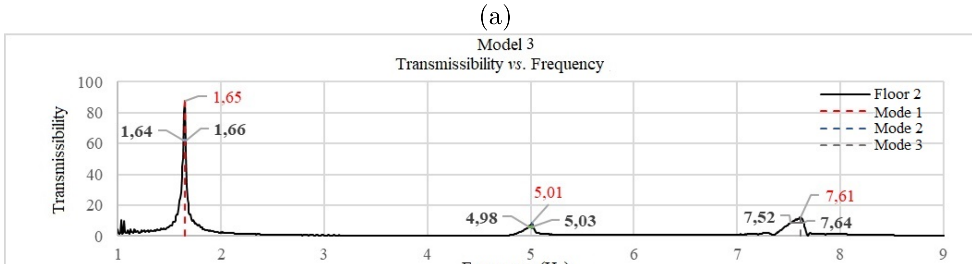


(b)

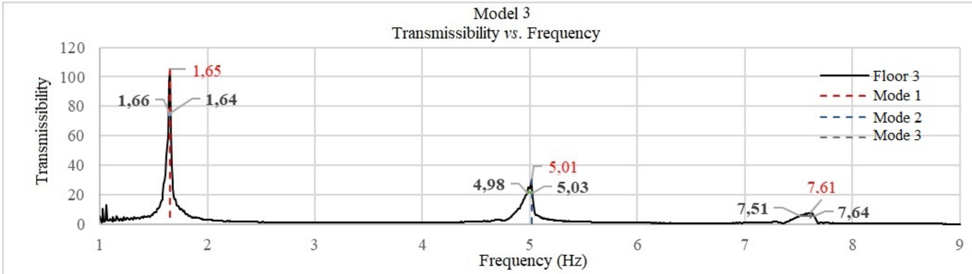
Figure 12. Frequency and bandwidth for model 2 (a) floor 1 and (b) floor 2



(a)



(b)



(c)

Figure 13. Frequency and bandwidth for model 3 (a) floor 1, (b) floor 2 and (c) floor 3

3.1.1. Frequencies and damping factor for model 1

Figure 11 shows the plots that enabled obtaining the theoretical frequency and the damping factor, and Table 4 displays these values.

The peak of Figure 11 shows that the experimental frequency for model 1 is 4.28 Hz.

Table 4. Frequency and damping for model 1

	ATH Dynamic responses	Exp.		Damping
Mode	f (Hz)	f (Hz)	% error	ξ (%)
1	4,58	4,28	6,55	1,44

3.1.2. Frequencies and damping factor for model 2

Figures 12 (a) and (b) show the plots that enabled obtaining the frequencies and damping factors for the two modes, and Table 5 presents the results.

The peaks of Figures 12 (a) and (b) are the experimental frequencies, which for mode 1 is 2.61 Hz and for mode 2 6.99 Hz.

Table 5. Frequency and damping for model 2

	ATH Dynamic responses	Exp.		Damping
Mode	f (Hz)	f (Hz)	% error	ξ (%)
1	2,61	2,60	0,38	0,57
2	6,53	6,99	7,07	0,85

3.1.3. Frequencies and damping factor for model 3

Figures 13 (a), (b) and (c) show the plots that enabled obtaining the frequencies and damping factors for the 3 modes, and Table 6 presents the results.

The peaks of Figures 13 (a), (b) and (c) are the experimental frequencies, which for mode 1 is 1.65 Hz, for mode 2 5.01 Hz and for mode 3 7.61 Hz.

Table 6. Frequency and damping for model 3

	ATH Dynamic responses	Exp.		Damping
Mode	f (Hz)	f (Hz)	% error	ξ (%)
1	1,91	1,65	13,61	0,55
2	5,25	5,01	4,57	0,49
3	7,36	7,6	3,26	0,85

3.2. Experiment 2. Dynamical response of the model to floor acceleration

The dynamical response was obtained when subjecting the experimental models to a scaled seismic record. The seismic record used was El Centro and its scaling was carried out using the software of the Shake Table II; the record was scaled based on maximum displacement of 4 cm.

The results presented in the following correspond to absolute accelerations, since these are the ones provided directly by the accelerometers, and they were compared with the absolute accelerations obtained from the ATH Dynamic response program, based on expression (19):

$$\ddot{x} = \ddot{u} + \ddot{x}_o \tag{19}$$

Where $\ddot{x}$ is absolute acceleration, $\ddot{u}$ relative acceleration and $\ddot{x}_o$ acceleration in the base.

3.2.1. Base acceleration record for Model 1: El Centro earthquake

Figure 14 shows the experimental acceleration and the theoretical acceleration for model 1, and Table 7 the maximum theoretical and experimental accelerations.

Table 7. Maximum accelerations for model 1

Acceleration (m/s^2) Floor 1		
Theoretical	Experimental	% error
8,707	7,657	13,711

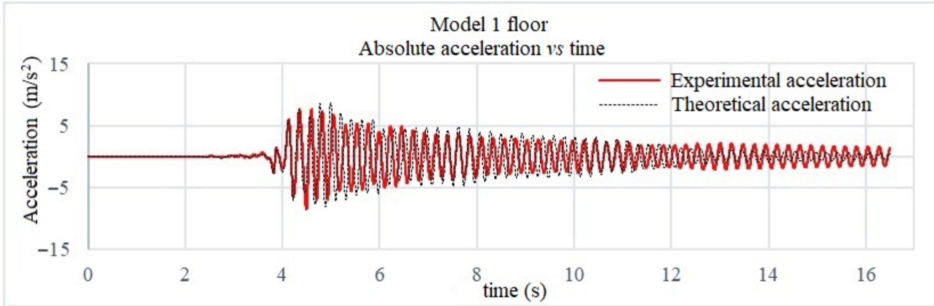


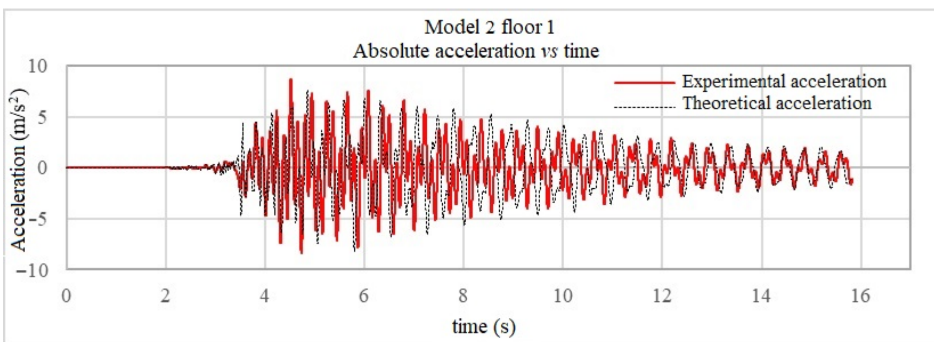
Figure 14. Experimental and theoretical absolute acceleration for model 1, with acceleration in the base of the El Centro earthquake

3.2.2. Base acceleration for Model 2: El Centro earthquake

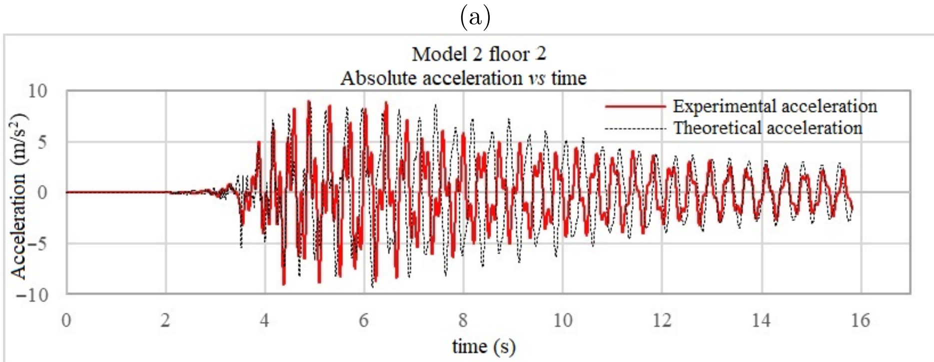
Figures 15 (a) and (b) show the experimental acceleration and the theoretical acceleration for model 2, and Table 8 the maximum theoretical and experimental accelerations.

Table 8. Maximum accelerations for model 2

	Acceleration (m/s^2)		
	Theoretical	Experimental	% error
Piso 1	7,585	8,678	12,586
Piso 2	8,989	9,021	0,353



(a)



(b)

Figure 15. Experimental and theoretical absolute acceleration for model 2, with acceleration in the base of the El Centro earthquake: (a) floor 1, (b) floor 2

3.2.3. Base acceleration for Model 3: El Centro earthquake

Figures 16 (a), (b) and (c) show the experimental acceleration and the theoretical acceleration for model 3, and Table 9 the maximum theoretical and experimental accelerations.

Table 9. Maximum accelerations for model 3

	Acceleration (m/s^2)		
	Theoretical	Experimental	% error
Piso 1	5,225	5,033	3,824
Piso 2	5,297	5,698	7,043
Piso 3	5,967	5,845	2,095

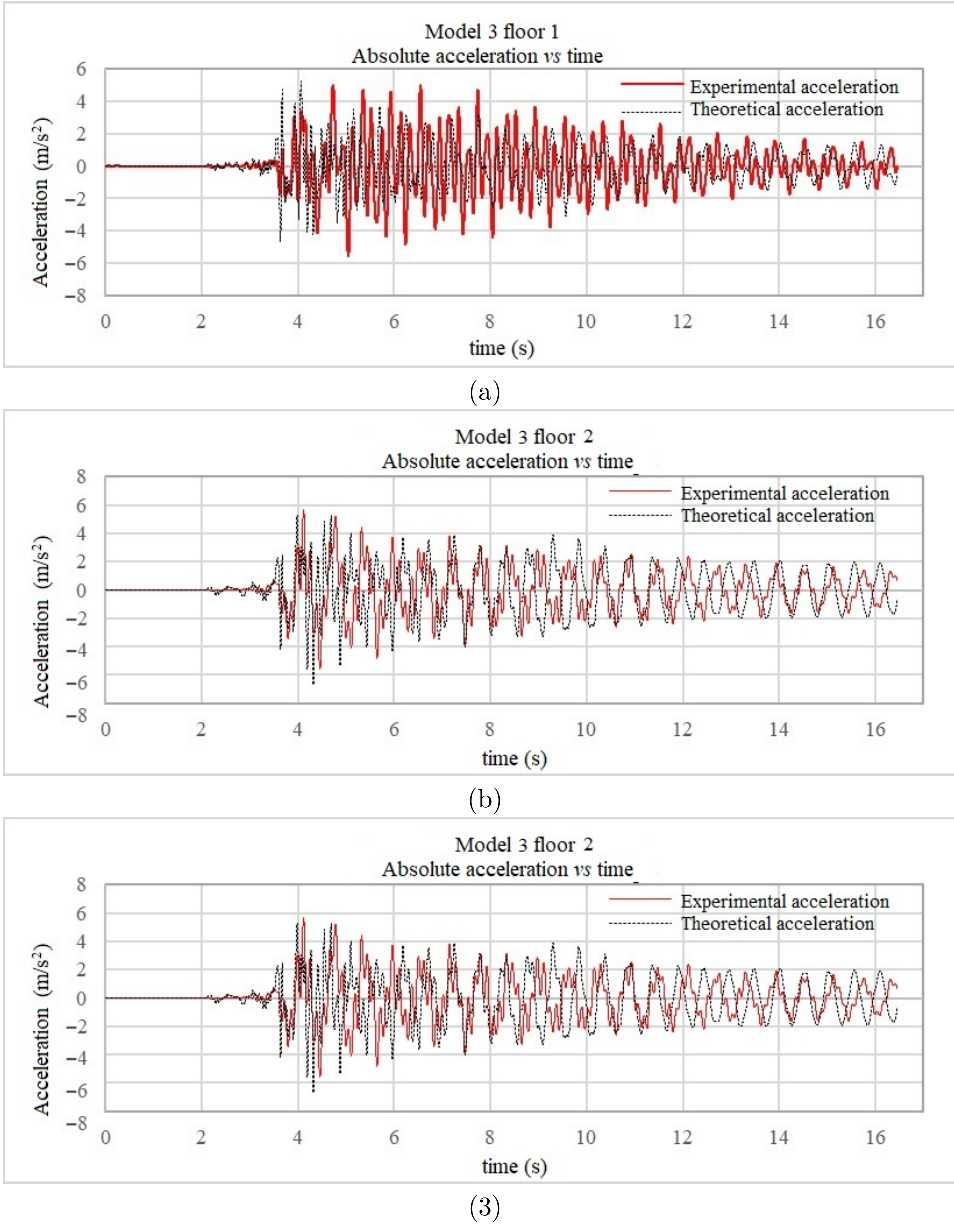


Figure 16. Experimental and theoretical absolute acceleration for model 3, with acceleration in the base of the El Centro earthquake: (a) floor 1, (b) floor 2, (c) floor 3

When graphically comparing the models, it was observed that the experimental acceleration has larger amplitude in most of the cases, however, in general the shapes of the experimental and theoretical plots are similar.

4. Conclusions

Based on the experimental results and the theoretical results of the models, it may be argued that the method which uses the Fourier transform to obtain the frequencies is appropriate, because the results are similar between experimental and theoretical frequencies.

The damping factor obtained by means of the bandwidth is consistent in terms of the values obtained per floor, since each floor provided similar values of damping factor per mode, and such factors were useful for theoretical modeling, and thus being able to obtain the damping matrix and the dynamical responses.

The experimental modeling of the model helped us to observe in a real manner how a model behaves when a sinusoidal acceleration or an acceleration of a scaled earthquake is applied, and enabled verifying the values obtained analytically with respect to the experimental ones.

As future work, it may be implemented to obtain the dynamical properties of three-dimensional systems that exhibit irregularity in plant and elevation.

A mismatch may be observed between experimental and theoretical responses regarding absolute accelerations, due to the consideration of the time 0 in which the experimental models were at rest.

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